

Intervention for Algebra I Module 1: Lessons





The Meadows Center
FOR PREVENTING EDUCATIONAL RISK
THE UNIVERSITY OF TEXAS AT AUSTIN
COLLEGE OF EDUCATION

Mathematics Institute for Learning Disabilities and Difficulties

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Module 1

Lesson 1

Lesson 1: Mathematical Symbols

Lesson Objectives	<p>Students will determine when symbols represent variables.</p> <p>Students will use the definition of <i>variable</i> to discriminate between examples and nonexamples of variables.</p>	
Vocabulary	variable: a symbol, usually a letter, that represents 1 value or a set of values	
Reviewed Vocabulary	none	
Instructional Materials	Teacher	Student
	<ul style="list-style-type: none"> Teacher Masters (pp. 1-10) Overhead/document projector 	<ul style="list-style-type: none"> Student Booklet (pp. 1-5) Variables foldable Whiteboard with marker

Engage Prior/Informal Knowledge

To open the lesson, present questions to activate students' background knowledge and prerequisite skills about different operational symbols used in mathematics.

Present the *Engage Prior Knowledge Practice Sheet* to students and provide each student with a whiteboard and marker.

As many of you know, mathematics is a special language that uses different types of symbols to represent the operations of addition, subtraction, multiplication, and division.

Draw an asterisk on the board.

For example, we use this symbol to indicate multiplication. It is an operational symbol.

Please take a few minutes to brainstorm, or think of, mathematical symbols for operations. Write the mathematical symbols on your whiteboard. You may use numbers to help demonstrate the mathematical symbols.

Give students 1–2 minutes to work with a partner.

Randomly call on students to hold up their whiteboards and share the symbols they wrote. They should say what the symbol is and what it represents.

Write the symbols on the *Engage Prior Knowledge Practice Sheet*. Be sure to include the basic operation symbols (+, −, ×, ÷). Students may write symbols that represent variables or symbols used for grouping. Acknowledge these symbols. If students do not contribute a symbol, ask probing questions, such as the following:

- What symbols indicate multiplication? (\times , \cdot , $3(4)$ means 3 times 4, $*$)
- What symbol shows that 2 quantities have the same value? ($=$)
- What symbols indicate division? (\div , $\frac{1}{2}$ means 1 divided by 2)

So far, we have discussed symbols that represent mathematical operations. In the language of mathematics, symbols can also represent *variables*.

Preview

This lesson will build on students' knowledge of the meaning and use of mathematical symbols, including variables.

Today, we will determine when symbols represent *variables*.

Demonstrate

1. Present and discuss the *Variables* foldable.

Distribute the *Variables* foldable to each student. Students will use the foldable throughout the module to organize information about the concept of variables. Write on a hard copy of the foldable throughout the lesson to model for students how to use it. Display the foldable for students to see, using an overhead projector or document projector.

Teacher Note

To maximize instructional time, cut and fold the foldable before teaching the lesson.

Today, we will use a foldable to help us organize information as we learn about *variables*. Open the foldable. Under the word “Variables,” there are sections: Examples, Characteristics, Definition, and Nonexamples.

“Examples” are illustrations that show the meaning of the word or concept being defined. “Nonexamples” are not illustrations of the word or concept being defined.

“Characteristics” are qualities that describe what we are defining. For example, tables have legs; therefore, legs are a

quality, or characteristic, of tables. “Definition” tells us the meaning of the word or concept.

Let’s use this foldable to find the meaning, or definition, of *variables*.

We will complete some of the sections of the foldable today to begin our discussion of *variables*.

2. Present and discuss examples of variables, using the *Variables* foldable.

Have students look at the situations presented in the Examples section of their foldable.

Look at the Examples section of your foldable. Remember that examples are items or words that illustrate the meaning of the word or concept being defined. We will first examine situations that contain examples of *variables*. Each of the following situations is an example of a *variable*.

Circle the variables in each of the situations listed on the *Variables* foldable.

I have circled each of the *variables* in each situation listed on your foldable. Circle these *variables* on your foldable.

Point to each of the situations listed and call on specific students to identify the variables.

Have students identify the characteristics of the variables circled in each situation.

We will examine each of these situations and look for characteristics that describe *variables*. Remember that characteristics are descriptions of what we are defining. Examine each of the circled *variables* and think about what they all have in common.

Pause for students to think.

Have students share what they notice. As the characteristics are discussed, make a bulleted list of them in the Characteristics section of the *Variables* foldable.

What do you notice about the circled items? (*answers may vary; they are all letters; they all represent a value or number*)

The first thing that we notice about each *variable* is that it is a letter. In the Characteristics section of your foldable, we will list the characteristics of *variables*. Write “letters” as 1 characteristic of *variables* in this section.

Ask students to think about what the letters stand for, or represent. Have students use the Think-Pair-Share routine for their observations to begin the discussion.

Look at the examples and think about what the letters stand for, or represent.

Pair with your neighbor to share what you think *variables* may stand for, or represent. One partner from each pair will be asked to share your ideas with the class.

After a few minutes, call on student pairs to share their thoughts about what variables stand for, or represent.

Discuss with students about how the variables in each situation are letters that represent 1 value or several values.

In the first 2 situations, the *variables* represent a single value or number. In the next 3 situations, the *variables* represent several values or numbers.

Teacher Note

Using variables to represent a set of values may be a new concept to students. This concept will be developed further in later lessons.

Write and have students write “1 value or number” and “represent several values or numbers” in the Characteristics section of the *Variables* foldable.

In the Characteristics section of your foldable, write “represent 1 value or number” and “represent several values or numbers.” Write these now.

Pause for students to write.

We will use these characteristics to build the definition of a *variable*.

3. Define “variable.”

Have students look at the characteristics of a variable that they have written on their foldable. Review the characteristics of variables with students. Using these characteristics, have students use the Think-Pair-Share routine to build a definition of a variable.

What are the characteristics of *variables*? (*letters or symbols that represent a single value or number, or that represent several values or numbers*)

Using the characteristics of *variables*, think about what the definition of a *variable* might be.

Pause for students to think.

Have students pair with a neighbor and discuss their definitions.

Pair with your neighbor to share what you think the definition of a *variable* might be and write your definition on the whiteboard. One partner from each pair will be asked to share your ideas with the class.

Pause for students to discuss.

Have student pairs share what they discussed by showing their whiteboard responses.

Teacher Note

Use a method to promote active student response and encourage all students to participate in class discussion. For example, randomly draw popsicle sticks or cards with students' names on them when selecting students to share their answers or reasoning.

Discuss and write the definition of the term “variable” in the Definition section of the *Variables* foldable. Use language from the students' definitions as it fits with the definition below. As you write the definition, have students copy the definition on their foldable.

In mathematics, a *variable* is a symbol, usually a letter, that represents 1 value or a set of values.

Write this definition in the Definition section of your Variables foldable. How does this definition compare to the definition you wrote with your partner? (*answers will vary*)

Have students repeat the definition of the term “variable.” Call on students randomly to provide the definition.

What is the definition of the term *variable*? (*a symbol, usually a letter, that represents 1 value or a set of values*)

Help students to connect their previous experience with variables to the use of variables in algebra. In the Examples section of the foldable, write and project on an overhead or document projector, “_____ + 5 = 8” and “ $x + 5 = 8$.” Have students copy the equations onto the Examples section on their foldable. Point to the equation that uses a blank to represent the missing value.

We want to make a connection between what we are learning now to something very similar in the earlier grades. You may have seen before an empty box or space to denote a missing

value, such as in this equation. This equation is: some number plus 5 is equal to 8.

Point to the equation that uses a variable to represent the missing value.

One use of a *variable* is to denote a missing or unknown value. In algebra, we use letters to represent *variables*, so blank + 5 = 8, becomes $x + 5 = 8$. Write these equations in the Examples section of your foldable.

4. Review examples and the definition of variables. Present and discuss situations that contain nonexamples of variables. Have students look at the Nonexamples section of the *Variables* foldable.

We've looked at examples of *variables*. Recall the definition of a *variable*. (*symbols, usually letters, that represent 1 value or a set of values*)

Now we will discuss nonexamples of *variables*. These are situations that use letters, but are not *variables*. Remember that nonexamples do not relate to the meaning of what you are defining – variables, in this case.

We will discuss 3 situations that use letters or symbols that are nonexamples of variables.

Discuss the first situation that represents a nonexample of a variable on the *Variables* foldable.


Look at the Nonexamples section of your foldable. The first situation that uses letters that are not *variables* is 7 cm. Although they are letters, “c” and “m” do not represent a value or set of values. Instead, these letters are an abbreviation for a unit of measurement. What unit of measurement do the letters “cm” represent? (*centimeters*)

Because these letters do not represent a value or set of values, this situation is a nonexample of a *variable*.

Have students provide other situations that use letters as abbreviations for units of measure. Write and have students write these situations in the Nonexamples section of the *Variables* foldable.

What is another nonexample, or situation where letters are used as abbreviations? (*answers may vary*)

Write these situations in the Examples section of your foldable.

<p>Watch For</p> 	<p>Students often think letters that are used for other purposes are variables.</p> <ul style="list-style-type: none">• For example, students might think the letter “m,” when used as an abbreviation for the unit of meters, is a variable.• Students also might think letters used as labels, such as in geometric shapes, are variables.
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Discuss the second situation that represents a nonexample on the *Variables* foldable. Letters that represent labels are another nonexample. Point to the second situation on the *Variables* foldable, in which the vertices of a triangle are labeled A, B, and C.

Look at the second situation in the Nonexamples section of your foldable. This is a situation in which letters are not *variables* because the letters are used as labels. Specifically, we use letters to label a geometric shape.

In triangle ABC, letters label the vertices, or corners, of the triangle. These letters are not *variables* because they do not represent a value or set of values. Instead, they are labels that are used to name the triangle.

Discuss the third situation that represents a nonexample on the *Variables* foldable.

Look at the third situation on your foldable. This shows another mathematical situation that does not use variables.

Point to the less than symbol and discuss with students why this symbol is not a variable.

In this situation, we see a mathematical symbol. Does anyone know what this symbol represents? (*less than symbol*)

This symbol is used in mathematics to show that 1 value is less than another. We call it the “less than” symbol. This symbol does not represent a value or set of values, so this situation is a nonexample of a *variable*.

Recall that the 3 situations we just discussed are nonexamples of *variables*. (*abbreviations of units, labels on geometric shapes, inequality symbols*)

5. Present the *Demonstration Practice Sheet* to students.

Have students recall the definition of the term *variable*.

What is the definition of the term *variable*? (*a symbol that represents 1 value or a set of values*)

Give students time to look over and think about whether each situation presented on the *Demonstration Practice Sheet* is an example or a nonexample of a variable.

Think about whether each situation on the sheet is an example or a nonexample of a *variable*. Circle “Example” or “Nonexample” in each situation.

Discuss with students whether situation 1, $1\text{ g} = 1,000\text{ mg}$, uses a variable. Make your thinking visible to students by using a think-aloud strategy to model and display how to write a justification of why each situation is an example or a nonexample of a variable.

Look at situation 1. This situation tells us that $1\text{ g} = 1,000\text{ mg}$. A *variable* is a letter or symbol that represents a value or set of values. What do the letters “g” and “mg” represent? (*abbreviations for units of measure*)

We know that letters representing units of measurement are not examples of *variables*.

Because “g” is an abbreviation for grams, what do you think “mg” represents? (*milligrams*)

Because the letters “g” and “mg” do not represent a value or set of values, they are nonexamples.

Circle and have students circle “Nonexample” for situation 1. Write and have students write “g and mg are abbreviations for units of measure” in the space provided.

Circle the word “Nonexample” for situation 1. In the space provided, write “g and mg are nonexamples because they are abbreviations for units of measure.”

Discuss with students why situation 2 is an example of a variable and write the justification.

Look at situation 2. The situation is: $3x - 1 = y$. The letters x and y represent a set of values. Because the letters x and y represent a set of values, they are examples of *variables*.

Circle and have students circle “Example” for situation 2. Write and have students write “ x and y represent a set of values” in the space provided.

Circle the word “Example” for situation 2. In the space provided, write that x and y are examples of *variables* because the letters x and y represent a set of values.

Discuss with students why situation 3 is an example of a variable. Ask students questions to elicit verbal responses to check for their understanding of examples and nonexamples of variables.

Look at situation 3. This situation is a square with side lengths of 3 and a .

Do you see any letters in this situation? (*yes, a*)

What is the relationship between the sides of a square? (*the lengths are the same*)

Because we know that the sides of a square are the same length, what do you predict is the value of a ? ($a = 3$)

Let's examine the letter " a ." Does the letter " a " represent an example of a *variable*? (*yes*)

Circle and have students circle "Example" for situation 3.

The letter " a " represents a value, so it is an example of a *variable*. Circle "Example" for situation 3.

How do you know that a is an example of a variable? (*it represents the length of 1 side of the square*)

Write and have students write " x and y represent a set of values" in the space provided.

In the space provided, write " a is an example of a *variable* because the letter ' a ' represents 1 value."

Discuss with students why situation 4 is a nonexample of a variable. Ask students questions to elicit verbal responses to check for understanding.

Look at situation 4. This situation also shows a geometric shape. The geometric shape is a triangle with numbers and letters for its side lengths.

Do you see any letters in this situation? (*yes, c and m*)

Let's examine the letters " c " and " m ." Do you think that the letters " c " and " m " are examples or nonexamples of *variables*? (*nonexamples*)

Circle and have students circle "Nonexample" for situation 4.

In this situation, the letters " cm " represent the unit centimeters. This means that the lengths of the sides of the triangle are 3 centimeters, 4 centimeters, and 5 centimeters.

Because the letters “c” and “m” do not represent values, they are nonexamples of *variables*. Circle “Nonexample” for situation 4.

How do you know that the letters “c” and “m” are nonexamples of *variables*?? (*because “cm” is an abbreviation for a unit of measure*)

Write and have students write “cm is an abbreviation for a unit of measure” in the space provided.

In the space provided, write that this situation is a nonexample because “cm” is an abbreviation for centimeters.

6. Provide a summary of the content on the foldable.

To review the key ideas of the lesson, ask students questions about the definition, characteristics, examples, and nonexamples of variables.

What is the definition of a variable? (*a symbol that represents 1 value or a set of values*)

What are some characteristics of variables? (*letters that represent a single value or number, or that represent several values or numbers*)

What are some examples of variables? (*answers will vary; check that students are providing letters that represent 1 value or a set of values*)

What are some nonexamples of variables? (*answers will vary; check that students are providing letters or symbols that do not represent 1 value or a set of values*)

How do we know when letters represent variables? (*answers will vary; they represent a value or set of values*)

How do we know when letters or symbols don't represent variables? (*answers will vary; they do not represent a value or set of values*)

Practice

Guided Practice

1. Discuss with students whether each situation in Activity 1 of the *Practice Sheet* is an example or a nonexample of a variable.
2. Make your thinking visible to students by using a think-aloud strategy to model a written justification of why each situation is an example or a nonexample of a variable.

Pair Practice

1. Have students work in pairs to determine whether each situation in Activity 2 is an example or a nonexample of a variable.
2. Have students use a think-aloud strategy to write a justification of their reasoning with their partners. Encourage students to use correct mathematical language with their partners.
3. Allow time for student pairs to present their reasoning to a neighbor pair. Each pair should take a turn justifying their response. Encourage students to use correct mathematical language in their presentation. Use questions such as the following to elicit detailed responses:
 - Does the letter represent a value or set of values? How do you know?
 - What does the letter represent?

Independent Practice

1. Have students work independently to fill in the blank for item 1 on the *Independent Practice Sheet* and then determine whether each situation presented in item 2 is an example or a nonexample of a variable.
2. Have students share their answers and their reasoning with the group. Provide corrective feedback using mathematical language from the lesson. Have students mark the total number correct at the top of their page.

Closure

Review the key ideas. Have students provide examples from the lesson. Have students discuss the following questions:

- In algebra, what do variables represent?
- What is an example of a variable?
- What is a nonexample of a variable?
- How can you tell whether a situation is an example of a variable?
- How can you tell whether a situation is a nonexample of a variable?

Module 1: Variables

Lesson 2

Lesson 2: Variables as Fixed Unknowns

Lesson Objectives	<p>Students will use variables as fixed unknowns to write and solve 1-step algebraic equations.</p> <p>Students will use precise vocabulary to communicate mathematical thinking about solving 1-step algebraic equations that contain variables as fixed unknowns.</p>	
Vocabulary	fixed unknown: a variable that represents a single value that makes an equation true	
Reviewed Vocabulary	product, sum, variables	
Instructional Materials	Teacher	Student
	<ul style="list-style-type: none"> Teacher Masters (pp. 11-22) Overhead or document projector 	<ul style="list-style-type: none"> Student Booklet (pp. 7-12) Variables foldable Whiteboard with marker

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet* as part of Engage Prior/Informal Knowledge.

Engage Prior/Informal Knowledge

To open the lesson, present questions to activate students' background knowledge and prerequisite skills about variables. Give each student a whiteboard and have students work with a partner to write the definition of the term "variable."

Last time, we discussed different types of symbols, including variables. As a class, we examined several examples and nonexamples of variables.

Work with a partner to write the definition of variable.

Pause for students to work.

What definition did you and your partner write? (*a symbol, usually a letter, that represents one value or set of values*)

Remember, not all letters are used as variables. Sometimes, letters are used as labels or abbreviations for units of measurement.

Discuss problem 1 from the *Cumulative Review Practice Sheet*.

In problem 1, you are asked to determine which of the 2 situations is an example of a variable. In choice A, is the letter *w* used as a variable? (yes) How do you know? (*"w" stands for the value of the length of the side, the letter is italicized*)

Why is answer choice B not the correct answer? (*because the letters A, B, C, and D are labels for the rectangle, not variables*)

Discuss problem 2 from the *Cumulative Review Practice Sheet*.

In problem 2, you are asked to determine which of the 2 situations does not use a variable. Which situation does not use a variable? (B)

Why is choice B the correct answer? *(there are many symbols in choice B but none represent a variable)*

Remember, a variable is a symbol, usually a letter, that represents 1 value or set of values.

Collect student papers to determine who needs additional instruction.

Preview

This lesson will build on students' knowledge of the meaning and use of mathematical variables as fixed unknowns.

Today we will use variables to write and solve 1-step algebraic equations for a *fixed unknown* value.

Demonstrate

1. Introduce variables as fixed unknowns and discuss the meaning of "fixed."

Have students take out their *Variables* foldable. Point to the section on your *Variables* foldable labeled "fixed unknown."

In mathematics, variables are used in many different situations. We will examine each of these situations and keep track of them on our Variables foldable. Your Variables foldable has 3 different sections – 1 for each situation in which we will use variables. The first situation that we will examine is when a variable is a *fixed unknown*.

Have students discuss the meaning of "fixed" in the context of mathematics variables.

Looking at the term "fixed unknown," what does the word "fixed" mean to you? *(answers may vary; something was broken but now is repaired)*

When we say that something is “fixed,” we can also be describing something that does not change. For example, if I have a fixed salary, this means that my salary does not change.

2. Present and discuss examples of a fixed unknown.

Provide an example to illustrate the definition. Write and have students copy the example inside the fixed unknown section of the *Variables* foldable.

We are going to examine an example of a variable as a *fixed unknown*. The equation $h + 3 = 5$ includes an example of a *fixed unknown*. Open the *fixed unknown* section on your Variables foldable. Write this example in the example section of the *fixed unknown* flap on your Variables foldable.

What value for the variable h makes this a true statement? (2)

Why does the variable h have to be 2? ($2 + 3 = 5$; no other value makes the equation true)

Would any other value for h make the equation true? (no)

Provide a second example of a fixed unknown. Write and have students copy the example in the example section inside the fixed unknown section on the *Variables* foldable.

Another example of a *fixed unknown* is the variable m in the equation $2m = 16$. Write this example in the example section inside of the fixed unknown flap on your Variables foldable.

What value of m makes the equation true? (8)

Why does the value of the variable m have to be 8? ($2 * 8 = 16$)

Would any other value for m make the equation true? (no)

Because the variable m can only be one value to make the equation true, m is an example of a variable as a *fixed unknown*.

3. Define “fixed unknown.”

State and write the definition of “fixed unknown” in the definition section inside of the fixed unknown section on your *Variables* foldable. Have students copy the definition inside the definition section of the fixed unknown section on their *Variables* foldable.

In the definition section of your Variables foldable, write the definition of a variable as a *fixed unknown*. In an equation, a variable is a *fixed unknown* when it represents a single value that makes an equation true.

Have students use the definition to examine the variables written in the examples section of the fixed unknown on their *Variables* foldable.

This means that in an equation, a variable as a *fixed unknown* can only be one value for the equation to be true.

Looking at the two examples we have written, $b + 3 = 5$ and $2m = 16$, do the variables b and m represent a single value? (yes)

Are the values, $b = 2$ and $m = 8$, the only numbers that make each equation true? (yes)

Then based on the definition, the variables b and m are *fixed unknowns*.

Have students repeat the definition of a variable as a fixed unknown.

What is the definition of *fixed unknown*? (a variable that represents a single value that makes an equation true)


Have students work with a partner to state the definition of “fixed unknown” in their own words. Then have student pairs write their definition on a whiteboard.

Work with your partner to explain or define, in your own words, a variable as a *fixed unknown*. Write your explanation on your whiteboard. Provide three examples of equations with variables that are fixed unknown.

Pause for students to work. Have student pairs share their examples and explanations.

What examples did you and your partner generate? (*answers will vary*)

How did you and your partner explain a variable as a *fixed unknown*? (*answers will vary*)

**Watch For**

Students may have difficulty with the concept of a variable due to the different situations in which variables are used. Emphasize that using a variable as a fixed, unknown value is just one of the many different situations in which we will see variables.

4. Use number puzzle problems to write equations and determine the value of fixed unknowns.

Introduce students to number puzzles on the *Demonstration Practice Sheet* by explaining what is written in the blanks in the number puzzle at the top of the sheet.

Look at the Demonstration Practice Sheet. We are going to work with number puzzles to examine variables as *fixed unknowns*. These number puzzles organize numbers, their products, and their sums into separate boxes.

Explain to students the organization of the quadrants in the number puzzle on the *Demonstration Practice Sheet*. Write, and have students write what is written in the blanks of the number puzzle on the *Demonstration Practice Sheet*. Point to each blank before you write “number,” “sum,” and “product.”

At the top of the Demonstration Practice Sheet there is a blank number puzzle. We will complete the blanks on the puzzle so you know how to use the puzzle. In the left and right sections of the number puzzle in the blanks we write “number.”

The bottom section of the number puzzle is the sum of the 2 numbers. Write “sum” in the blank at the bottom of the number puzzle.

The top section of the number puzzle is the product of the 2 numbers. Remember, the product is the solution to a multiplication problem. Write “product” in the blank at the top of the number puzzle.

Write and have students write in the equation blanks to demonstrate how to use the number puzzle to write the equations on the *Demonstration Practice Sheet*.

The lines to the side set up how the equations are written. In the first 2 blanks of the top equation, we write “number.” In the last blank, we write “product” because we are multiplying the 2 numbers. So, our equation is: number times number equals product.

In the first 2 blanks of the bottom equation, we write “number.” In the last blank, we write “sum” because we are adding the 2 numbers. So, this equation is: number plus number equals sum.

Teacher Note
Allow students to use a calculator for computations as necessary. It is important that students understand the concept and do not get lost in the calculations.

Now, let’s move on to problem 1.

Write and have students write as you model the process to use the number puzzle for problem 1 on the *Demonstration Practice Sheet*.

In problem 1, on the Demonstration Practice Sheet, we are given the numbers. We have to find the product and sum to complete the puzzle.

Write and have students write “ $(4)(7) = 28$ ” in the equation blanks. Write “28” in the product section of the number puzzle on the *Demonstration Practice Sheet*.

**In order to complete the number puzzle we write the numbers “4” and “7” in the number blanks in both of the equations. Looking at the top equation, what is the product of 4 and 7?
(28)**

Because the product of 4 and 7 is 28, write the product, “28,” in the last blank of the top equation. Also write “28” in the top section of the number puzzle.

What is the sum of 4 and 7? (11)

Because the sum of 4 and 7 is 11, we write “11” in the last blank of the bottom equation. Also write “11” in the bottom section of the number puzzle.

Guide students through problem 2 on the *Demonstration Practice Sheet* to figure out the missing value in the puzzle. Explain that because the missing value, or variable, is a fixed unknown, there is only 1 value that will make each equation true in the number puzzle. Point to the variable y .

The number puzzle in problem 2 has a missing value. The missing value represents a *fixed unknown* value for the variable y . We will use the number puzzle to find the value of the *fixed unknown* variable.

Write and have students write in the equation blanks to help them find the value for the fixed unknown variable in problem 2 on the *Demonstration Practice Sheet*.

Based on the number puzzle in problem 2, the product of 5 and y is 30. In the blanks for the top equation, we write “5,” “ y ,” in the number blanks of the equation and “30” in the product blank of the equation to show this relationship.

The sum of 5 and y is 11. In the blanks for the bottom equation, we write “5,” “ y ” in the number blanks of the equation, and “11” in the sum blank of the equation.

Pause for students to work.

Look at the top equation. To find the value of y , we ask ourselves, what number multiplied by 5 equals 30? (6)

Look at the bottom equation. To find the value of y , we ask ourselves, what number added to 5 equals 11? (6)

Because the sum of 5 and 6 is 11 and the product of 5 and 6 is 30, the number puzzle solution is $y = 6$. There is no other value for y that would make each equation true. Therefore the variable y is a *fixed unknown* value.

Guide students through problem 3 on the *Demonstration Practice Sheet* to figure out the missing value in the number puzzle. Explain that because the missing value, or variable, is a fixed unknown, there is only 1 value that will solve the number puzzle.

The number puzzle in problem 3 has a missing value. We will use the number puzzle to find the value of the *fixed unknown* variable p .

Write and have students write in the equation blanks to help them find the value for the fixed unknown variable in problem 3 on the *Demonstration Practice Sheet*. Use specific problems to elicit verbal responses to check for understanding.

Based on the number puzzle, the product of 9 and p is 54. What do we write in the blanks of the top equation? $((9)(p) = 54)$

Write $(9)(p) = 54$ in the top equation.

The sum of 9 and p is 15. What do we write in the blanks of the bottom equation? $(9 + p = 15)$

What *fixed unknown* value of p makes the product of the 2 numbers equal 54? (p represents 6)

What *fixed unknown* value of p makes the sum of the 2 numbers equal 15? (p represents 6)

In other words, the *fixed unknown* value of p that makes each equation true is 6.

Guide students through problem 4 on the *Demonstration Practice Sheet* to figure out the missing value in the number puzzle.

The number puzzle in problem 4 has a missing value that is represented by the variable h .

Write and have students write in the equation blanks to help them find the value for the fixed unknown variable in problem 4 on the *Demonstration Practice Sheet*. Use specific problems to elicit verbal responses to check for understanding.

The product of h and 3 is -12. What do we write in the blanks for the top equation to show the product? ($we\ write\ (h)(3) = -12$)

The sum of h and 3 is -1. What do we write in the blanks for the bottom equation to show the sum? ($we\ write\ h + 3 = -1$)

What value of h makes the sum of the 2 numbers equal -1? ($h = -4$)

What value of h makes the product of the 2 numbers equal -12? ($h = -4$)

Is h a variable that is a *fixed unknown* value? (yes)

Why? (-4 is the only value that makes each equation true)

5. Summarize variables as fixed unknowns.

Explain to students that the variables in the number puzzles are fixed unknown values because only 1 value makes the equations true.

For the last 3 examples, we wrote equations to find the value of each variable. Because only 1 value makes each equation true, the variables are *fixed unknowns*.

Practice

Pair Practice

1. Have students work with a partner to solve the problems on the *Practice Sheet* by determining the value for the variable as a fixed unknown in the number puzzles and completing the equations to reinforce their solutions.
2. Have student pairs share the reasoning for their answers to the class. Ask questions such as the following to elicit detailed responses:
 - What value of the variable makes the equation a true statement? Is this the only value for the variable that makes the equation true?
 - How do you know that the variable is a fixed unknown?
 - What value of the variable makes the product of [number] and the variable equal to [number]?
 - What value of the variable makes the sum of [number] and the variable equal to [number]?

Error Correction Practice

1. Have students examine the 2 solution strategies presented and determine which strategy is incorrect. Have students justify why the solution strategy that they chose is incorrect.
2. Have students share the reasoning for their answers to the class.

Independent Practice

1. Have students determine the variables as fixed unknowns in the number puzzles on the *Independent Practice Sheet* and justify their reasoning.
2. Have students share their answers with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson. Have students discuss the following question:

- How do you know whether a variable is a fixed unknown?

Module 1: Variables

Lesson 3

Lesson 3: Variables in a Generalized Pattern, Part I

Lesson Objectives	<p>Students will use variables to write algebraic generalizations of patterns.</p> <p>Students will use correct mathematical language to discuss patterns and algebraic generalizations.</p>	
Vocabulary	generalization: a statement that describes a pattern	
Reviewed Vocabulary	fixed unknown, variable	
Instructional Materials	Teacher	Student
	<ul style="list-style-type: none"> Teacher Masters (pp. 23-34) Overhead or document projector 	<ul style="list-style-type: none"> Student Booklet (pp. 13-18) Variables foldable Whiteboard with marker

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet* as part of Engage Prior/Informal Knowledge.

Engage Prior/Informal Knowledge

Review variables as fixed unknowns. Give student pairs a whiteboard and marker to write their responses to the review problems. Have students work with a partner to define variables as fixed unknowns.

What is a variable? (*a symbol that represents 1 value or a set of values*)

A variable is a symbol that represents 1 value or set of values. In the last lesson, we worked with variables as fixed unknowns. Discuss with your neighbor the definition of a variable as a fixed unknown and write it on your whiteboard.

Pause for students to work.

What definition did you and your partner write? (*a variable that represents only 1 value that makes an equation true*)

Discuss the problems on the *Cumulative Review Practice Sheet*. First, determine whether problem 1 uses a variable.

Look at problem 1 on the Cumulative Review Practice Sheet. Which of the given situations uses a variable? (B)

How do you know that y in answer choice b is a variable? (*y represents the value of the length of the side*)

How do you know that the other answer choices are not examples of variables? (*answers may vary*)

Determine the value of the fixed unknown variable b in problem 2 on the *Cumulative Review Practice Sheet*.

Look at the number puzzle in problem 2. This number puzzle has a missing (unknown) value that is represented by the variable b .

Remember, the top number, -48, is the product of 6 and b , and the bottom number, -2, is the sum of 6 and b . What equations could you write to represent these relationships?
 $(6)(b) = -48$ and $6 + b = -2$

What fixed unknown value of b makes the product of the 2 numbers equal to -48 and the sum of the 2 numbers equal to -2? (b represents -8)

How did you arrive at -8? (answers will vary; “guess and check” as well as “solving” are acceptable)

Variables represent 1 value or a set of values. How do you know when a variable represents a fixed unknown? (when a variable can represent only 1 value to make an equation true, the variable is a fixed unknown)

Today we will learn about when a variable represents more than 1 value.

Preview

This lesson will build on students’ knowledge of the meaning and use of mathematical variables in generalized patterns.

Today we will use variables to write algebraic *generalizations* of patterns in numerical equations.

Demonstrate

1. Introduce and explain variables in generalizations. Present and discuss examples of variables in a generalization.

Point to the next section labeled “in a generalization” on the *Variables* foldable.

We have been using our Variables foldable to keep track of the different situations in mathematics in which we see variables.

Today, we will use variables in a *generalization*. Look at your foldable and find the next section labeled “in a *generalization*.”

Provide students with an example of a variable in a generalization to help illustrate the concept. Have students write the example in the example section of their *Variables* foldable.

In the example section of the *generalization* flap of your Variables foldable, we will write an example of a variable in a *generalization*.

The first example of a variable in a *generalization* is the equation $a + 0 = a$. Write the equation in the example section of your foldable.

We can create this *generalization* from the following pattern.

Write and have students write the following equations that demonstrate the pattern in the example section of the foldable.

Write the following 4 equations in the example section. We will use this pattern of equations to create the *generalization*: $a + 0 = a$.

- $1 + 0 = 1$
- $2 + 0 = 2$
- $3 + 0 = 3$
- $4 + 0 = 4$

We can use variables to make a *generalization* that describes the pattern and works for a large set of numbers. Any number plus 0 is equal to itself. In the *generalization* $a + 0 = a$, the variable a can represent any number.

To write the *generalization* of the pattern, we must identify what we always do in each equation.

Looking at all 4 equations, we take a number and add 0 to the number. When we add 0 to the number, the result is always the same number.

Teacher Note

Some students struggle with writing or too much information on a page. An option is to highlight the areas of the equation that stay the same instead of circling them.

Circle and have students circle the “+ 0” in each equation.

We will circle + 0 in all equations because that is the operation and value that occurs in each of the equations.

Pause for students to work.

The number that we add to 0 is different in each equation. Because the numbers are different, we use a variable to represent the set of values that change from equation to equation. The variable a is used in the *generalization* of the pattern.

Is there a value for a that would not work in our *generalization*, $a + 0 = a$? (no)

How do you know? (if 0 is added to a number, that number does not change)

2. Define “generalization.”

State and write the definition of a generalization in the definition section of the *Variables* foldable. Have students copy the definition in the definition section of their *Variables* foldable.

Because we will use variables in a *generalization* today, we need to define what a *generalization* is.

In the definition section of your foldable, write the definition of a *generalization*. *Generalizations* are statements that describe a pattern. A pattern is a way to arrange numbers and variables to follow a rule or rules.

An example of a *generalization* in the real world may help us understand the definition. I notice that maple trees have green leaves, oak trees have green leaves, and pecan trees have green leaves. My *generalization* is that ALL trees have green leaves. This is a statement that describes a pattern that I see.

Let's try another generalization. If a Chihuahua barks, a beagle barks, and a German shepherd barks, what *generalization* describes this pattern? (*all dogs bark*)

Have students repeat the definition of a generalization.

What is the definition of a *generalization*? (*a statement describing a pattern*)

Looking back at our example on the foldable, the *generalization*, $a + 0 = a$, is a mathematical statement describing the pattern we see in the numerical equations.

3. Guide students through developing algebraic generalizations of the pattern in problem 1 of the *Demonstration Practice Sheet*.

Write and have students write as you model the process to create an algebraic generalization of the pattern. Guide students through the reasoning by using specific questioning.

The first problem on the Demonstration Practice Sheet shows equations that demonstrate a pattern. We create a *generalization* by asking ourselves, “What do we always do mathematically in each equation?”

Point to the number “1” in each equation. Write and have students write in the blank what happens in each equation.

Looking at all 4 equations, we are multiplying by 1 in each equation. On your Demonstration Practice Sheet, the first

prompt is “In each equation we _____.” Write “multiply by 1” in the blank.

Write and have students write on the *Demonstration Practice Sheet* the variable, what the variable represents, and why.

Next, we use a variable to represent the values that change in the pattern of equations. On your *Demonstration Practice Sheet*, the second prompt is “The variable _____ represents _____ because _____.” Write in the first blank the variable “*a*.” Remember we can use any letter as a variable. Looking at the equations, what does the variable *a* represent? (*the numbers 2, 3, 4, and 5*)

Write in the blank after “represents” the numbers “2, 3, 4, and 5.”

The reason *a* represents the numbers 2, 3, 4, and 5 is because the numbers change from equation to equation. Therefore, the variable *a* represents the set of values 2, 3, 4, and 5. In the blank after “because,” write “the numbers change from equation to equation.”

Now we need to use the variable *a* to create a *generalization* of the pattern that describes what happens for all equations.

A number multiplied by 1 is equal to itself.

Watch For



Students may have difficulty using variables to write a generalization or rule. Guide students to understand the generalization in words before translating into an equation. For example, emphasize that any number multiplied by 1 is equal to that same number before writing the equation “ $(n)(1) = n$.”

Write and have students write a generalization for the pattern, using a variable on the *Demonstration Practice Sheet*.

Because we always multiply by 1 in the pattern, it must be included in the *generalization* of the pattern.

We picked a as the variable in the *generalization* of the pattern. What does the variable represent? (*any number that makes the equation true; for this pattern so far it is 2, 3, 4, and 5*)

So, the equation is $(a)(1) = a$. Check your *generalization* by using 2 more numbers of your choosing. Select any number you can think of and substitute it for a .

Teacher Note

Demonstrate and encourage the use of negative numbers here. Using parenthesis to substitute values will help to reinforce that the variable really does represent ANY number. For example, students can check using -6 by writing $(-6)(1) = -6$.

Pause for students to work.

Is the *generalization* true for the numbers you used? (*yes*)

Write the *generalization* “ $(a)(1) = a$ ” on the blank provided for the *generalization* for this pattern.

We also need to translate this into words. How would you phrase what is happening in this *generalization*? (*answers will vary*)

Write “a number, a , multiplied by 1 will equal itself” in the last blank in problem 1.

4. Guide students through developing algebraic generalizations of the pattern in problem 2 of the *Demonstration Practice Sheet*.

Write and have students write as you model the process to create an algebraic generalization of the pattern. Guide students through the reasoning by using specific questioning.

The second problem on the Demonstration Practice Sheet shows 3 equations that demonstrate a pattern. Look at each equation. What are we always doing mathematically in the equations? (*subtracting to equal 0*)

Point to the subtraction and “= 0” in each equation. Write and have students write in the blank what happens in each equation.

In all 3 equations, we are subtracting 2 numbers and they equal 0. On your Demonstration Practice Sheet, in the blank of the first prompt write, “subtract to equal 0.”

Write and have students write on the *Demonstration Practice Sheet* the variable, what the variable represents, and why.

Next we use a variable to represent the values that change in the pattern of equation. On your Demonstration Practice Sheet, what letter should we use to represent the values that change? (*answers will vary, for the script we will use b*)

Looking at the pattern, what values will the variable b represent? (*the numbers 6, 7, and 8*)

Write the number “6, 7, and 8” in the blank after “represents.”

Why does b represent 6, 7, and 8? (*those are the values that change from equation to equation*)

Now we need to use the variable b to create a *generalization* of the pattern that describes what happens for all equations.

Write and have students write a generalization of the pattern, using a variable on the *Demonstration Practice Sheet*.

What are we always doing in this pattern? (*we are subtracting a number from itself and it equals 0*)

If the variable b represents the values that change from equation to equation, how would we write this *generalization*?
($b - b = 0$)

Check your *generalization* by using 2 more numbers of your choosing. Select any number you can think of and substitute it for b .

Pause for students to work. Write and have students write “ $b - b = 0$ ” and “a number subtracted from itself will equal 0” on the *Demonstration Practice Sheet*.

For the values you choose, was the *generalization* true? (yes)

Write “ $b - b = 0$ ” on the blank for the *generalization* for the pattern on your *Demonstration Practice Sheet*.

How would you phrase what is happening in this *generalization*? (answers will vary)

We write “a number subtracted from itself will equal 0” in the last blank in problem 2.

5. Guide students through developing algebraic generalizations of the pattern in problem 3 of the *Demonstration Practice Sheet*.

Write and have students write as you model the process to create an algebraic generalization of the pattern. Guide students through the reasoning by using specific questioning.

Problem 3 of the *Demonstration Practice Sheet* shows 3 equations that demonstrate a pattern. What is the first step to writing the *generalization* of this pattern of equations? (we have to find what is always happening mathematically)

What are we always doing mathematically in the equations?
(dividing by 1)

Write “dividing by 1” in the blank after “In each equation we...”

What is the next step to finding the *generalization*? (*define a variable, identify what it represents*)

What variable will we use for the *generalization*? (*answers will vary, for the script we will use c*)

Write “ c ” in the blank for the variable.

What does the variable c represent? (*the numbers 2, 4, and 8*)

After “represents,” write the values “2, 4, and 8” on the blank provided.

Why does the variable c represent the set of numbers 2, 4, and 8? (*because the numbers are the values that are changing from equation to equation*)

Write “the numbers change from equation to equation” on the blank provided on your Demonstration Practice Sheet.

Have students write the generalization of the pattern on whiteboards and show you their response.

I want you to predict what the *generalization* will be for this pattern. On your whiteboard, use the variable c to write a *generalization* of the pattern that you see in each equation.

Pause for students to work.

Show me the *generalization* that you wrote on your whiteboard to describe the pattern in problem 3. ($\frac{c}{1} = c$)

Write and instruct students to write the equation for the generalization of the pattern on the *Demonstration Practice Sheet*.

The *generalization* of the pattern in problem 3 can be written with the equation $\frac{c}{1} = c$.

Check your *generalization* by using 2 more numbers of your choosing. Select any number you can think of and substitute it for b .

Pause for students to work. Write and have students write “ $c/1 = c$ ” and “a number divided by 1 will equal itself” on the *Demonstration Practice Sheet*.

For the values you chose, was the *generalization* true? (yes)

Write the *generalization* “ $\frac{c}{1} = c$ ” in the blank provided for the pattern in problem 3.

How would you phrase what is happening in this *generalization*? (answers will vary)

Write “a number divided by 1 will equal itself” in the last blank in problem 3.

6. Provide a summary of the content on the foldable and on the *Demonstration Practice Sheet*.

To review the key ideas of the lesson, ask students questions about using variables in a generalization.

In your own words, what is the definition of a *generalization*?
(a statement that describes a pattern)

Practice

Pair Practice

1. Have students work with a partner to answer the problems on the *Practice Sheet* by matching the correct generalization with the corresponding set of numerical equations. Tell students to be ready to justify their reasoning.
2. Have student pairs explain their reasoning for their answers to the class. Encourage students to use correct mathematical language. Ask probing questions such as the following to elicit detailed responses:
 - What is the first step to finding the generalization of a pattern?
 - What do we always do mathematically in each equation?
 - What is the next step to finding the generalization of a pattern?

- What does the variable represent? How do you know?
- How do you write the generalization of this pattern? How do you know?

Error Correction Practice

1. Have students examine the 2 solution strategies presented and determine which strategy is incorrect. Have students justify why the solution strategy that they chose is incorrect.
2. Have students share the reasoning for their answers to the class.

Independent Practice

1. Have students match the correct generalization to the corresponding set of numeric equations.
2. Have students share their answers with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students describe variables in a generalization in their own words. Give students time to think of a response based on the examples in the lesson.

- We have written the definition of a *generalization* and looked at examples of variables used in a *generalization*. How would you explain, in your own words, a variable in a *generalization*?

Module 1: Variables

Lesson 4

Lesson 4: Variables in a Generalized Pattern, Part II

Lesson Objectives	<p>Students will use variables to represent numbers in a generalized pattern from linear geometric and tabular patterns.</p> <p>Students will create and use representations to organize, record, and communicate mathematical ideas to peers and teachers.</p>	
Vocabulary	<p>common difference: the change in output values from 1 step to the next step is the same for a set of data</p> <p>input: the set of numbers or values that are used to generate the output values</p> <p>output: the set of numbers or values obtained by applying a rule or generalization to each input value</p>	
Reviewed Vocabulary	generalization, variable	
Instructional Materials	Teacher	Student
	<ul style="list-style-type: none"> Teacher Masters (pp. 35-46) Overhead or document projector Square colored tiles 	<ul style="list-style-type: none"> Student Booklet (pp. 19-24) Square colored tiles (1 set with 3 different colors per student pair) Whiteboard with marker Colored pencils

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet* as part of Engage Prior/Informal Knowledge.

Engage Prior/Informal Knowledge

Lead a discussion to activate students' background knowledge and prerequisite skills about using variables as fixed unknowns and in generalizations of patterns.

Have students use their *Variables* foldable to aid them in this discussion.

In previous lessons, we learned about 2 different situations that use variables.

The first situation in which we have used variables is as a fixed unknown. How do you know if a variable is a fixed unknown? *(the variable represents 1 single value)*

The second situation that we have seen variables in is in a generalization of a pattern. What is the definition of a generalization? *(a statement describing a pattern)*

Discuss the problems on the *Cumulative Review Practice Sheet*.

Look at problem 1 of the Cumulative Review Practice Sheet. The number puzzle in this problem has a missing value that is represented by the variable n . What does the value of n have to be to make the equations from our number puzzle work? *($n = 6$)*

Can the variable n represent any other value that would make the equation true? *(no, 6 is the only one that works)*

Because variable n represents the only value that makes our equations true, it is a fixed unknown.

Look at problem 2. This problem shows 3 equations that demonstrate a pattern.

What mathematical operations are repeated in each of the equations? (*subtracting to equal 0*)

If the variable h represents the values that change from equation to equation, how would we write this generalization?
($h - 0 = h$)

We have examined patterns in numerical equations and have used variables to write generalizations of these patterns.

Preview

This lesson will build on students' knowledge of the meaning and use of mathematical variables in generalized patterns.

Today we will use sets of table values to examine and describe tile design patterns in a generalization.

Demonstrate

1. Guide students through developing an algebraic generalization for the pattern in problem 1 of the *Demonstration Practice Sheet*.

Have students complete the *Demonstration Practice Sheet* as you model the process of analyzing a tile design pattern.

Look at problem 1 on the Demonstration Practice Sheet. There are pictures representing stages of a tile design. Each stage represents more tiles being added to the design.

Have students identify the number of tiles in each stage by calling on specific students. Point to the tile design for Stage 1.

Look at Stage 1 of the tile design. There are 4 total tiles used to build the design in Stage 1. How many tiles are used to build the design in Stage 2? (8 tiles)

How many tiles are used to build the design in Stage 3? (12 tiles)

Discuss with students how the total number of tiles changed from Stage 1 to Stage 2.

How did the total number of tiles change from Stage 1 to Stage 2? (answers may vary; there is an increase in 4 tiles)

Watch For



Students may have difficulty seeing the differences among each stage. Have students color the new tiles in each stage a different color to make the amount of tiles added in each stage more visible.

Write and have students write “+4 tiles” in the blank between Stage 1 and Stage 2 of the tile designs for problem 1.

There were 4 tiles added between Stage 1 and Stage 2. Write “+4 tiles” in the blank between Stage 1 and Stage 2 of the tile designs.

How many tiles were added to Stage 2 to create Stage 3? (4 tiles added)

Write and have students write “+4 tiles” in the blank between Stage 2 and Stage 3 of the tile designs for problem 1.

There were 4 tiles added between Stage 2 and Stage 3. Write “+4 tiles” in the blank between Stage 2 and Stage 3.

If this pattern were to continue, how many tiles do you predict will be added between Stage 3 and Stage 4? (4 tiles)

If we add 4 tiles to Stage 3 to create Stage 4, predict what the total number of tiles will be for Stage 4. (16 tiles)

How do you know? (answers may vary; 12 tiles plus 4 tiles equals 16 tiles)

2. Use the information in the tile design to fill in the “Total” in the table provided for problem 1 on the *Demonstration Practice Sheet*. Instruct

students to also fill this information in on the tables of their *Demonstration Practice Sheets*.

Discuss with students how to use the table to keep track of the stage number and the total number of tiles.

Look at the table for problem 1. We will use this table to record information about the tile design.

Point to the first column labeled “(Stage) n .” Have students locate this column on their tables.

The first column is used to record the stage number. We will use the variable, n , to represent the stage number.

Point to each value of n and connect it to the stage number.

When $n = 1$, this represents Stage 1 of the tile design. When $n = 2$, this represents Stage 2 of the tile design. What does $n = 3$ represent? (Stage 3)

What does $n = 4$ represent? (Stage 4)

Point to the last column labeled “Total.” Have students locate this column on their tables.

The last column in the table is used to record the total number of tiles in each stage.

Write and have students write the total number of tiles for each stage in the last column.

In Stage 1, there were 4 total tiles in the tile design, so when $n = 1$, there is a total of 4 tiles. We will write “4” in the first row.

What was the total number of tiles in Stage 2? (8)

What was the total number of tiles in Stage 3? (12)

What did we predict the total number of tiles would be in Stage 4? (16)

Write and have students write “+4/1” in the blanks to the right of the table showing the change in tile totals from stage to stage.

Next to the column for the total tiles there are blanks. We are going to record how the tile totals change from 1 stage to the next stage.

The change in tiles from Stage 1 to Stage 2 is +4 tiles/1 stage. In the blank we write “+4/1” because the tiles increased by 4, while the stage increased by 1.

The change we write in the blank for Stages 2 to 3 is +4/1 because the tile totals increase by 4, while the stage again increased by 1.

We have predicted that the tile totals in Stage 4 would be 16. How do you think we record this change in the blank provided? (+4/1)

Teacher Note

It is important to write “ $\frac{4 \text{ tiles}}{1 \text{ stage}}$ ” and discuss the unit rate in this manner to prepare students for (and review of) discussions of slope. This will also address the fact that the common difference remains the same even when terms are “skipped.”

Once each blank is filled with $\frac{4 \text{ tiles}}{1 \text{ stage}}$, a discussion can be held about $\frac{4}{1} = 4$ to fill in the common difference blank prior to writing the generalization.

Define input and output and discuss with students which set of values is the input, and which is the output. Write “input” above the first column and “output” above the last column.

We have two sets of numbers, or values. The first set is the stage numbers. The second set of values is the total number of tiles. We will define the first set of values, the stage number, as our *input* values. The *input* values are the set of numbers that are used to generate the *output* values.

Output is the set of numbers or values obtained by applying a rule or generalization to each *input* term. In this tile design situation, the total number of tiles is the source of the *output* values.

We need to find the generalization, or rule, that will give us the total number of tiles for any stage number. The first step to finding a generalization will be to find the *common difference*.

3. Define “common difference.”

Discuss the definition of common difference.

We have already found the *common difference* for problem 1. The *common difference* is 4. How did we get this number?
(answers will vary)

We found a *common difference* of 4 by looking at how the *output*, or total numbers, changed for each stage.

The *output* values changed by the same amount for each stage.

The type of change we have seen in this pattern is called a *common difference*.

When the change in *output* values from one step to the next is the same for a set of data, we can call the result the *common difference*.

Write and have students write “+4 tiles per stage” in the space labeled “Common Difference” for problem 1.

What is the *common difference* in problem 1? (+4 each time)

The *common difference* for problem 1 is positive 4 because we had an increase of 4 tiles per stage. Write the *common difference* in the space provided.

4. Use the common difference to describe the relationship between the stage number and the total number of tiles. Draw and have students draw circles or counters in the Thinking Process column of the table to represent the total number of tiles for each stage.

We will use the Thinking Process column to help us write the generalization. In this column, we will draw counters, or circles, to represent the number of tiles for each stage.

Draw and have students draw 4 counters in the first row of the Thinking Process column.

There are 4 tiles in Stage 1, so we will draw 4 counters for Stage 1. Do this now.

How many counters should we draw for stage 2? (8 counters)

Draw and have students draw 8 counters in the second row of the Thinking Process column. Draw, and have students draw counters for Stages 3 and 4 in the third and fourth rows of the Thinking Process column in the table for problem 1.

Draw counters for Stages 3 and 4 in the third and fourth rows of the Thinking Process column in the table for problem 1.

Discuss with students that the grouping of counters is based on the common difference and the term. The number of counters in each group is the same as the common difference, and the number of groups circled matches the stage, or term number. Circle and have students circle counters into groups.

We will put our counters into equal groups by circling them. Use the colored pencils to circle the groups. The number of counters in each group must be the same as the *common difference*.

What is the *common difference*? (+4)

Because the *common difference* is $+4$, we will circle groups of 4 counters in each stage.

For Stage 1, we circle only 1 group of 4 because it is the first stage and there are only 4 counters to circle. Circle 1 group of 4.

For Stage 2, there are 8 counters, so we will circle 2 groups of 4 counters.

Remember, because our *common difference* is 4, we are circling groups of 4. How many groups do we circle for Stage 3? (3 groups of 4)

How many groups of 4 do we circle for Stage 4? (4 groups of 4)

Have students verbalize a description of each group.

For Stage 1, there is 1 group of 4 counters. How would you describe the counters in Stage 2? (there are 2 groups of 4 counters)

How would you describe Stage 3 counters? (3 groups of 4)

How are Stage 4 counters different? (there are 4 groups of 4; there is 1 more group of 4)

Write and have students write the mathematical statements. Use these statements to compare similarities and differences.

In mathematics, what operation do we use when we add together equal groups? (multiplication)

Watch For



Students may not remember or recognize that adding together equal groups can represent multiplication. Use a simple example to help students see the connection.

To illustrate, write the following parenthetical equations on the whiteboard as needed.

When we add together equal groups, we use multiplication. For Stage 1, we can express the counters mathematically as “1(4).”


Stage 2 is expressed mathematically as “2(4).” Verbally, this means 2 groups of 4.

How can we express 3 groups of 4 mathematically? $(3(4))$

How can we express 4 groups of 4 mathematically? $(4(4))$

When we write a generalization, we want to state what mathematical operations we do each time in the pattern. Because we always multiply the stage number by 4, that must be in the generalization for the pattern. We describe the generalization by saying it is always 4 times the stage number, n .

The generalization of this pattern is written as “ $4n$.”

<p>Watch For</p> 	<p>Students may write the generalization as “$n4$” or “$n(4)$” instead of “$4n$.” Discuss with students that although $n(4)$ may have the same meaning as $4n$, it is an algebraic convention to write the coefficient before the variable.</p>
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Point to the “4” in the generalization. Have students recognize that the “4” in the generalization is the common difference.

Look at our generalization. Where does the number “4” in $4n$ come from? *(it is the common difference)*

Where does the “ n ” in $4n$ come from? *(it is the stage number)*

This generalization says that for any stage number, n , we can find the total number of tiles by multiplying n by 4.

5. Guide students through developing algebraic generalizations of the pattern in problem 2 of the *Demonstration Practice Sheet* by repeating the process outlined above.

Walk students through the thought process using specific questions, such as the following, to elicit verbal responses from students and check for their understanding:

- What is the difference in output values between Stage 2 and Stage 1? Between Stage 2 and Stage 3? Do the input values change by a constant amount?
 - Are the changes in output values for each stage the same?
 - What do you think the increase of tiles will be from Stage 3 to Stage 4?
 - What is the common difference?
 - How did you find the common difference?
 - How do we group the counters?
 - What does n represent?
 - What mathematical operations do we see in each row of the table?
 - What should be included in the generalization?
6. Provide a summary of the content and process used in the lesson.

To review the key ideas of the lesson, ask students questions about the common difference and a generalization.

How do you find the common difference in a table? (*find the change or difference in the output values and the change or difference in the input values*)

In your own words, what is a generalization? (*answers will vary; a statement that describes a pattern*)

Practice

Activity 1: Guided Practice

1. Guide students through building the generalization of the pattern on the first page of the *Practice Sheet*.
2. Use specific questions, such as the following:
 - What is the common difference?
 - What does n represent?
 - What mathematical operations do we see in each row of the table?
 - What should be included in the generalization?
3. Encourage students to use correct mathematical language.

Activity 2: Pair Practice

1. Have students work with a partner to answer the problem on the second page of the *Practice Sheet*.
2. Label students A and B. Have student A complete row 1 and explain to student B.
3. Have student B complete and explain row 2 to student A.
4. Have student pairs continue alternating for rows 3 and 4.
5. Have student pairs work together to explain and write the generalization row for the pattern. Encourage students to use correct mathematical language.
6. Have student pairs present their reasoning by explaining their answers to the class. Ask questions to elicit detailed responses. Encourage students to use correct mathematical language. Use probing questions to elicit detailed responses, such as the following, to check for student understanding:
 - What is the common difference?

- What does n represent?
- What mathematical operations do we see in each row of the table?
- What should be included in the generalization?

Independent Practice

1. Have students fill out the table, find the common difference, and use it to write a generalization for the pattern on the *Practice Sheet*.
2. Have students share their answers with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students provide examples from the lesson. Have students discuss the following questions:

- What is the common difference?
- How do you find the common difference?
- What is a generalization?
- How do we group the counters?

Module 1: Variables

Lesson 5

Lesson 5: Variables in a Generalized Pattern, Part III

Lesson Objectives	<p>Students will use variables to represent numbers in a generalized pattern from linear geometric and tabular patterns.</p> <p>Students will create and use representations to organize, record, and communicate mathematical ideas to peers and teachers.</p>	
Vocabulary	constant term: a term that contains no variable and does not change value	
Reviewed Vocabulary	common difference, generalization, variable	
Instructional Materials	Teacher	Student
	<ul style="list-style-type: none"> Teacher Masters (pp. 47-58) Overhead or document projector Colored pencils 	<ul style="list-style-type: none"> Student Booklet (pp. 25-30) Colored pencils (2–3 per student)

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet* as part of Engage Prior/Informal Knowledge.

Engage Prior/Informal Knowledge

Review students' understanding of "common difference" and the use of variables to write equations for numerical patterns.

Look at problem 1 of the Cumulative Review Practice Sheet. This problem shows 3 equations that demonstrate a pattern. We are asked to find the generalization of this pattern. What is a generalization? (*a statement that describes a pattern*)

In each equation, what are we always doing, mathematically? (*dividing a number by 1*)

What happens when you divide a number by 1? (*answers will vary; it is equal to the same number*)

Does this work for any number? (*yes*)

If the variable n represents the values that change from equation to equation, how would we write this generalization? (*$n/1 = n$*)

For the second problem, you had to find the correct generalization for the pattern in the table. We can do this by finding the common difference. What is the definition of "common difference"? (*the difference between output values when input values differ by 1*)

How do you find the common difference? (*answers will vary*)

What is the common difference? (*2*)

What was the correct generalization for the pattern in the table? (*$2n$*)

In this generalization, what does the 2 represent? (*the common difference*)

Today we will continue to work with patterns and tables and to use variables to write generalizations of those patterns.

Preview

This lesson will build on students' knowledge of the meaning and use of mathematical variables in generalized patterns.

Today we will use variables to represent numbers and write generalizations of geometric and table patterns.

Demonstrate

1. Guide students through developing an algebraic generalization for the pattern in problem 1 of the *Demonstration Practice Sheet*.

Have students complete the *Demonstration Practice Sheet* as you model the process of developing an algebraic generalization. Guide students through the reasoning, using specific questioning.

Look at problem 1 on the Demonstration Practice Sheet. There are pictures representing stages of a tile design. The number of tiles in each stage is recorded in the table. We will use this table to write a generalization. The first step is to find the change from each stage to the next. If they are all the same, then we know the table has a common difference. What is the common difference for this table? (+2)

How did you find the common difference? (*answers will vary*)

Write and have students write “+ 2/1” in the blank next to each arrow to indicate the common difference.

To write the generalization for any pattern, we need to start by finding the common difference. On your Sheet, write “+ 2/1” in the blanks next to the arrows to show the common difference.

Discuss with students the grouping of counters based on the common difference and the term number. The number of counters in each group is the same as the common difference, and the number of groups circled matches the stage, or term number.

Let's put our counters into equal groups by circling them. Use the colored pencils to circle the groups. The number of counters in each group will be the same as the common difference.

What is the common difference? (2)

Circle and have students circle counters in each row, creating groups based on the stage or term number on the *Demonstration Practice Sheet*.

Because the common difference is 2, we will circle groups of 2 counters in each stage.

For Stage 1, we circle only 1 group of 2 because it is the first stage.

For Stage 2, we circle 2 groups of 2. The number of counters circled matches the stage number.

Remember, because our common difference is 2, we are circling groups of 2. How many groups do we circle for Stage 3? (3 groups of 2)

How many groups of 2 do we circle for Stage 4? (4 groups of 2)

Have students verbalize a description of each group.

For Stage 1, there is 1 group of 2 counters. How would you describe Stage 2 counters? (there are 2 groups of 2 counters)

How would you describe Stage 3 counters? (3 groups of 2)

How are Stage 4 counters different? (there are 4 groups of 2)

Write and have students write the mathematical statements. Use these statements to compare similarities and differences.

In mathematics, what operation do we use when we add together equal groups? (*multiplication*)

Watch For



Students may not remember or recognize that adding together equal groups can represent multiplication. Use a simple example to help students see the connection.

To illustrate, write the following parenthetical equations on your copy of the *Demonstration Practice Sheet*, “ $1(2) + 0$,” “ $2(2) + 0$,” “ $3(2) + 0$,” and “ $4(2) + 0$.”

When we add together equal groups, we use multiplication. For Stage 1, we can express the counters mathematically as “ $1(2)$.” Are there any extra counters outside of the circled group? (*no*)

There were no extra counters. In mathematics, we would represent this with $+ 0$. Because there are no extra counters outside of the circled groups, we write “ $+ 0$.”

Stage 2 could be expressed as “ $2(2) + 0$.”

How can we express Stage 3? ($3(2)+0$)

How can we express Stage 4? ($4(2)+0$)

Because we always multiply by 2 and then add 0, this must be in the generalization for the pattern.

What changes? (*the number we are multiplying*)

What number is multiplied by 2? (*the stage number, n , term number*)

What do we use to represent a value that changes? (*a variable*)

Explain the use of the word “term” when pictorial representations are not present.


When looking at patterns in mathematics, we usually use the letter n to represent the stage. The stage can also be called the term number. Recall using the word “term” when you worked with sequences and series. We also use the word “term” when there are no picture representations. This means that n represents the stage or term number in the pattern.

Write and have students write on the last line of the table, “ n groups of 2 plus 0 extra” and “ $2(n) + 0$ ” on the *Demonstration Practice Sheet*. The last line completes the problem, ending with the generalization of the pattern.

In the last box for the Term column, the variable, n represents any number. We need to describe the Thinking Process column, just like the other stages. This means that there are n groups of 2 plus 0 extra. Write this in the blanks of the last row.

Pause for students to complete.

We then write in the Total column, “ $2(n) + 0$.” Therefore, the generalization of this pattern is written as “ $2(n) + 0$ ” or just “ $2n$.”

 <p>Watch For</p>	<p>Students may write the generalization as “$n2$” or “$n(2)$,” instead of “$2n$.” Discuss with students that although $n(2)$ may have the same meaning as $2n$, it is an algebraic convention to write the coefficient before the variable.</p>
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2. Guide students through developing algebraic generalizations of the pattern in problem 2 of the *Demonstration Practice Sheet*.

Write, and have students write, as you model the thinking process for developing an algebraic generalization of the pattern in problem 2.

For problem 2 on the Demonstration Practice Sheet, we have only the table of values to use to write the generalization of the pattern.

Have students find the common difference. Remind students to fill in the blanks next to the arrows to show the changes between the total values.

We need to start by finding the common difference. Be sure to write the change between totals in the blanks next to the arrows. What is the common difference in this pattern? (3)

Remind students that the number of counters in each group matches the common difference and the number of groups for each term matches the term number.

The counters for the total have been drawn to represent the pattern in the Thinking Process column. The next step in the thinking process is to circle equal groups of counters. Finding the common difference tells us how many counters will be in each group. How many counters will be in each equal group? (3)

Because the common difference is 3, we will circle groups of 3.

Circle and have students circle groups of counters based on the term number and the common difference on the *Demonstration Practice Sheet*.

Circle groups of 3 for each term. Remember, the number of groups circled will match the term number.

How many groups of 3 do we circle for the first term? (1 group of 3)

Are there any extra counters outside of the circled groups? (yes)

How many extra counters outside of the circled groups? (1 counter)

Have students repeat the process for second, third, and fourth terms. For each term, students should notice that 1 counter is not circled, or is left over.

How many extra counters are outside of the circled groups, or left over, in each term? (*1 counter*)

We had 1 extra counter for each term. Because this counter is outside of the groups, it is not multiplied. Instead, this extra counter is added.

Guide students through the process of writing the mathematical statements for each term based on the circled counters in each row.

How many groups of 3 do we have for first term? (*1*)

Was that all of the counters for first term? (*no*)

How many counters are not circled or were extra? (*1 counter*)

We have 1 group of 3 plus 1 extra counter. How can we express 1 group of 3 mathematically? (*$1(3)$*)

How do we express the 1 extra counter mathematically? (*$+ 1$*)

Fill in and have students fill in the equation “ $1(3) + 1$,” “ $2(3) + 1$,” “ $3(3) + 1$,” and “ $4(3) + 1$ ” in the Thinking Process column on the *Demonstration Practice Sheet*.

We can then fill in the mathematical statement by writing “ $1(3) + 1$.”

Teacher Note

Constant terms may add a level of difficulty for students. Emphasize that the $+ 1$ corresponds to the 1 counter left over.

How should we describe the counters for the second term? (*2 groups of 3 plus 1 extra*)

How do we express that description mathematically? $(2(3) + 1)$

Repeat this process for the third and fourth terms.

Point to the mathematical statements and have students describe what always happens, mathematically, in each statement.

Looking at the mathematical statements, the term number is multiplied by the common difference and then we are adding the *constant term*.

What do we always do, mathematically, in each statement?
(multiply by 3 and then add 1)

What changes in each mathematical statement? *(the number multiplied by 3)*

What do we call the number that is multiplied by 3? *(the term number)*

What do we use to represent the value that changes? *(variable)*

We will represent the term number with the variable n . Because multiplying by 3 and adding 1 occurs for all statements, it must be included in the generalization of the pattern.

Write and have students write on the last line of the table, “ n groups of 3 plus 1 extra” and “ $3(n) + 1$.” The last line completes the problem, ending with the generalization of the pattern.

Fill in the Thinking Process column at the bottom. How do we write the pattern using the n th term? *(n groups of 3 plus 1 extra)*

Pause for students to work.

Now, how do we write the generalization of this pattern? $(3n + 1)$

State and label the constant term in the generalization on the *Demonstration Practice Sheet*. Have students do the same.

Looking at the generalization, $3n + 1$, the number 1 is called a *constant term*. A *constant term* is a term that does not contain a variable and does not change value. On the generalization, write the label “constant term” below “+ 1.”

Have students restate the definition of a constant term.

What is the definition of a *constant term*? (*a term that does not contain a variable and does not change value*)

Don’t let the word “term” here be confusing. Remember that we can use some words in more than one way. Here, we are talking about a term in an expression.

Above we are discussing terms in a series of items and term refers to the stage.

3. Provide a summary of the content and process used in the lesson.

To review the key ideas of the lesson, ask students questions about the common difference, generalization, and constant term.

How do you find the common difference in a table? (*find the change or difference in the output values when the input values differ by 1*)

When writing a generalization of a pattern, how do you know which values are multiplied and which values are added? (*the common difference is multiplied by the term number because that is the number of counters we circle in groups, while any counters outside of the groups, the constant term, are added*)

What is a *constant term*? (*the number that is added; no variables and does not change*)

Practice

Activity 1: Guided Practice

1. Guide students through building the generalization of the pattern on the first page of the *Guided Practice Sheet*.
2. Use specific questions, such as the following:

- What is the first step to writing a generalization of a pattern?
 - What is the common difference?
 - How do we group the counters? How is the term number related to the grouping?
 - How do we describe the counters after grouping?
 - Are there any counters not circled in the groupings?
 - Is there a constant term?
 - How do we write this in mathematics? What do we always do mathematically in the statements?
3. Encourage students to use correct mathematical language.

Activity 2: Pair Practice

1. Have students work with a partner to answer the problem on the second page of the *Guided Practice – Pair Practice Sheet*.
2. Label students A and B. Have student A complete row 1 and explain to student B.
3. Have student B complete and explain row 2 to student A.
4. Have student pairs continue alternating for rows 3 and 4.
5. Have student pairs work together to explain and write the generalization row for the pattern. Encourage students to use correct mathematical language.
6. Have student pairs present their reasoning by explaining their answers to the class. Ask questions to elicit detailed responses. Encourage students to use correct mathematical language, such as common difference, constant term, and variable. Use probing questions to elicit detailed responses to check for their understanding, such as the following:
 - How did you find the pattern?
 - What does n represent?
 - What is the constant term in the generalization?

Independent Practice

1. Have students complete the table and write the generalization of the pattern.
2. Have students share their answers with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Give students time to think of a response to the following, based on the examples from the Demonstrate section.

- How do you use the common difference in the generalization of a pattern?
- How do you use the *constant term*?

Module 1: Variables**Lesson 6**

Lesson 6: Variables in a Generalized Pattern, Part IV

Lesson Objectives	<p>Students will use variables to represent numbers in a generalized pattern from linear geometric and tabular patterns.</p> <p>Students will create and use representations to organize, record, and communicate mathematical ideas to peers and teachers.</p>	
Vocabulary	No new words are introduced.	
Reviewed Vocabulary	common difference, constant term, generalization, input, output	
Instructional Materials	Teacher	Student
	<ul style="list-style-type: none"> Teacher Masters (pp. 59-70) Overhead or document projector 	<ul style="list-style-type: none"> Student Booklet (pp. 31-36)

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet* as part of Engage Prior/Informal Knowledge.

Engage Prior/Informal Knowledge

Review the concepts of common difference and constant terms, using problems from the *Cumulative Review Practice Sheet*.

The first problem of the Cumulative Review Practice Sheet asks you to find the common difference. The common difference is + 4. How do you find the common difference? (*answers may vary; 8 tiles – 4 tiles = 4 tiles, etc.*)

Which of the following is the correct generalization? (C $4n$)

The second problem asks you to find the correct generalization for the pattern in the table. The answer was D $3n + 2$. In this generalization, what does the 3 represent? (*the common difference*)

What does the + 2 represent? (*the constant term; the value added*)

Today we will continue to work with table patterns and write generalizations for the patterns.

Preview

This lesson will build on students' knowledge of the meaning and use of mathematical variables in generalized patterns.

In today's lesson, we will continue to write generalizations to express the patterns we find in tables.

Demonstrate

1. Guide students through developing an algebraic generalization of the pattern in problem 1 of the *Demonstration Practice Sheet*.

Have students complete the *Demonstration Practice Sheet* as you review and model the process to create a generalization of the pattern in the table. Guide students through the reasoning, using specific questions to check for their understanding of generalizations of patterns in a table. Display the *Demonstration Practice Sheet* using an overhead projector or document projector for students to see.

Previously, we wrote generalizations of patterns from a table. We will continue to work with patterns in a table, but we will pay closer attention to the term number.

Have students find the common difference and write it in the blanks next to the arrows in problems 1 of the *Demonstration Practice Sheet*.

We need to start by finding the common difference. How do we find the common difference? (*answers may vary; $5 - 3 = 2$, $3 + a \text{ number} = 5$*)

What is the common difference in this pattern? (*2*)

Write “+2” in the blanks next to the arrows.

Have students draw counters for rows 3 and 4 in the Thinking Process column.

Complete the Thinking Process column in the table. Draw the missing counters for the third and fourth term. How many counters do you draw for the 3rd term? (*7 counters*)

Pause for students to work.

Circle and have students circle groups of counters based on the term number. Remind students that the number of counters in each group matches the common difference and that the number of groups circled matches the term number.

Next, circle equal groups of counters, based on the common difference and the term number. How many counters will be in each equal group? (2)

Teacher Note

This activity is not recommended for problems that involve a non-integer constant difference. This will involve circling fractional groups of counters and may lead to more confusion for students.

Because the common difference is 2, the equal groups will contain 2 counters each. Circle groups of 2, but remember that the number of groups must match the term number. For example, how many groups of 2 do we circle for term 1? (1 group of 2)

Continue circling the counters for the remaining terms.

Pause for students to work.

Are there extra counters outside of the circled groups? (yes)

How many extra counters are outside of the circled groups? (1)

Is there always 1 counter outside of the circled groups? (yes)

Recall that this extra counter, when written in mathematical statements, is called a constant term because it does not change value and does not contain a variable.

Guide students through the process of writing the mathematical statements for each term in the Thinking Process column.

We will describe the counters in words to help write the mathematical statements. We have to count the number of groups, the number of counters in each group, and the number of extra counters.

Write and have students write “ $1(2) + 1$ ” in the Thinking Process column.

For the first term, we describe the group as 1 group of 2 plus 1 extra. This means in mathematics we write “ $1(2) + 1$ ” in the Thinking Process column.

How would you describe the second term? (*2 groups of 2 plus 1 extra*)

How would you write this in mathematics? ($2(2) + 1$)

Write and have students write “ $2(2) + 1$ ” in the Thinking Process column.

Write “ $2(2) + 1$ ” in the Thinking Process column.

Repeat the process with students for the third and fourth terms. For each term students should notice the pattern of multiplying by 2 and adding 1. Point to the mathematical statements and have students describe what mathematical **operations** are **repeated** in each statement.

What mathematical operations are repeated in each statement?
(*multiply the term number by 2 and then add 1*)

Because we always multiply the term number by 2 and then add 1, we can now write a generalization of this pattern. The last row is the n^{th} term, which represents any term number for the pattern. This is the general term for the pattern. How do you describe the n th term? (*n groups of 2 plus 1 extra*)

How is the n^{th} term written mathematically? ($n(2) + 1$)

Write and have students write the generalization “ $2n + 1$ ” on the *Demonstration Practice Sheet*.

Remember that in mathematics, the convention, or standard way, of writing this statement is $2n + 1$. Now you can find any total for any given term.

Watch For



Students may write the generalization as “ n^2 ” or “ $n(2)$ ” instead of “ $2n$.” Discuss with students that although $n(2)$ may have the same meaning as $2n$, it is an algebraic convention to write the coefficient before the variable.

2. Guide students through developing algebraic generalizations of the pattern in problem 2 of the *Demonstration Practice Sheet*.

Have students complete the *Demonstration Practice Sheet* as you review and model the thinking process to create a generalization of the pattern in the table. Utilize specific questions to check for students’ understanding of generalizations of patterns in a table.

Based on the previous example, what is the first step to writing a generalization for the pattern we see in a table? (*find the common difference*)

Have students find the common difference and write it in the blanks next to the arrows in problem 2 of the *Demonstration Practice Sheet*.

Find the common difference and write it in the blanks next to the arrows. What is the common difference? (+ 2)

What does the common difference tell us? (*how many counters are in each group*)

Ask questions to guide students through the grouping process in the Thinking Process column.

Next, we draw counters for each of the terms in the Thinking Process column. How will we know how many counters to draw in each row? (*as many as are listed in the Total Column*)

Draw the number of counters for each total, or output value.

Circle and have students circle groups of 2 counters based on the term number. Remind students that the number of groups of 2 circled should match the term number.

Let's circle groups of 2 for each term in the Thinking Process column.

In the first row, even though more counters could be grouped, we circle only 1 group because it is the first term. Remember, the number of circled groups must match the term number.

For the next row, we circle 2 groups of 2 counters because it is the second term.

How many groups of 2 do we circle for the third term? (3)

How do we know? *(because it is the third term and the term number matches the number of groups circled)*

How many groups of 2 do we circle for the fourth term? (4)

Guide students through the process of writing the mathematical statements for each term in the Thinking Process column.

In each term, how many counters are outside of the circled groups, or extra? (3)

For the first row, how would you describe the group and extra counters? *(1 group of 2 plus 3 extra)*

How do we express 1 group of 2 plus 3 extra mathematically?
(1(2) + 3)

Write and have students write "1(2) + 3" in the Thinking Process column.

Write "1(2) + 3" in the Thinking Process column.

Now, look at the second term. How do you write the second term as a mathematical statement? *(2(2) + 3)*

Write and have students write "2(2) + 3" in the Thinking Process column.

Write "2(2) + 3" in the Thinking Process column.

Repeat this process with students for the third and fourth terms.

Have students describe the mathematical operations repeated across the mathematical statements.

Look at all of the mathematical statements. We want to identify the pattern, so we can use variables to write the generalization.

What mathematical operations are repeated? (*multiply the term number by 2 and then add 3*)

Because we always multiply the term number by 2 and then add the constant term 3, these mathematical operations must be included in the generalization.

How do we represent the term number for the generalization? (*the variable n*)

Write and have students write " $2n + 3$ " in the last line of the table on the *Demonstration Practice Sheet*. The last line will complete the problem, ending with the generalization of the pattern.

What must the generalization for the pattern include? (*multiply by 2 and add 3*)

What is the generalization for this pattern? ($2n + 3$)

Problem 2 was different from previous examples because the constant term was greater than the common difference. So, we had to be aware that the number of groups must match the term number.

Teacher Note

Extension: The counter activity can still be used on problems with negative values for the multiplier and/or constant term by adding in 0 pair counters. The addition of 0 pair counters will allow the correct number of positive or negative counters to be circles into groups. Explain to students how the addition of 0 pairs does not change the value.

3. Provide a summary of the content and process used in the lesson.

To review the key ideas of the lesson, ask students questions about the common difference, generalization, and constant term.

How do you find the common difference in a table? (*find the change or difference in the output values when the input values differ by 1*)

When writing a generalization of a pattern, how do you know what values are multiplied and what values are added? (*the common difference multiplies the term number because that is the number of counters we circle in groups and the counters outside of the groups is the number that is added*)

What is the constant term? (*the number that is added; no variables and does not change*)

Practice

Teacher Note

If students need additional support, change this activity from Pair Practice to Guided Practice.

Pair Practice

1. Have students work with a partner to complete the *Practice Sheet*.
2. Label students as A and B. Have student B complete row 1 and explain to student A.
3. Have student A complete row 2 and explain to student B.
4. Have student pairs continue alternating roles for rows 3 and 4.
5. Have pairs work together to explain and write the generalization row for the pattern. Encourage students to use correct mathematical language.

6. Have pairs explain their answers to the class. Use probing questions such as the following to elicit detailed responses:
 - What is the common difference?
 - How did you find the pattern?
 - What is the constant term in the generalization?
 - What does n represent?

Error Correction Practice

1. Have students examine the solution strategy presented and determine why the strategy is incorrect.
2. Have students share the reasoning for their answers to the class.

Independent Practice

1. Have students complete the table and write the generalization of the pattern.
2. Have students share their answers with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Give students time to think of a response to the following, based on examples from the lesson.

- Why is it important to write a generalization for a pattern?
- How do you use the common difference in the generalization of a pattern?
- How do you use the constant term?

Module 1: Variable**Lesson 7**

Lesson 7: Variables used in Verbal Translations, Part I

Lesson Objectives	<p>Students will define variables and translate verbal descriptions to equations from contextual situations.</p> <p>Students will use precise mathematical language when translating verbal descriptions to equations from contextual situations.</p>	
Vocabulary	No new words are introduced.	
Reviewed Vocabulary	difference, product, quotient, sum, variable	
Instructional Materials	Teacher	Student
	<ul style="list-style-type: none"> Teacher Masters (pp. 71-80) Overhead/document projector Colored pencils/markers 	<ul style="list-style-type: none"> Student Booklet (pp. 37-41) Colored pencils/markers Whiteboard with marker

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet* as part of Engage Prior/Informal Knowledge.

Engage Prior/Informal Knowledge

To open the lesson, lead a discussion to activate students' background knowledge and prerequisite skills of using variables to write generalizations of patterns.

Discuss problem 1 on the *Cumulative Review Practice Sheet*.

To answer the first problem on the Cumulative Review Practice Sheet, you had to choose the correct generalization of the pattern in the table. The first step is to find the common difference. What is the common difference for the first problem? (3)

How did you find the common difference? (answers will vary)

Look at each of the numerical statements in the table. What operations and numbers are the same for each statement? (multiply by 3)

We always multiplied the term number by 3 to get the total.

Which generalization shows that for any term number, n , we multiply n by 3? ($3n$)

What does the 3 in $3n$ represent? (the common difference)

Discuss problem 2 on the *Cumulative Review Practice Sheet*.

For the second problem, you had to choose the correct generalization of the pattern. Again, the first step is to find the common difference. How do you find the common difference? (answers may vary)

Look at each of the numerical statements in the table. What operations and numbers are the same for each statement?
(multiply by 3 and add 2)

We always multiplied the term number by 3 and then added 2 to get the total.

Which generalization shows that for any term number, n , we multiply n by 3, then add 2? ($3n + 2$)

Preview

This lesson will build on students' knowledge of the meaning and use of mathematical variables in contextual situations.

Today we will discuss variables in contextual situations and write equations that represent these situations.

Demonstrate

1. Guide students through the brainstorming process to create a translation reference sheet.

Write and have students write a list on their *Demonstration Practice Sheet* as a reference to translate word problems in the future. Using an overhead projector or document projector, display the *Demonstration Practice Sheet* for students to see.

Translating from a word problem to an equation can be difficult because of the many words that represent mathematical processes and operations. We will create a list of some of these words. We can use this list to help us read and understand word problems.

Hand out a whiteboard and whiteboard marker to each student to brainstorm a list of words that represent mathematical processes. Draw and have the students draw 5 columns labeled "Addition," "Subtraction," "Multiplication," "Division," and "Equal" on their whiteboards.

On your whiteboard, draw 5 columns and label them “Addition,” “Subtraction,” “Multiplication,” “Division,” and “Equal,” similar to your Demonstration Practice Sheet.

Pause for students to work.

Write and have students write the word “sum” under the Addition column as an example to start the brainstorming process.

You have 3 minutes to brainstorm and write all the words you can think of that mean the same thing as, addition, subtraction, multiplication, division, and equal in each column. For example, the word “sum” means the result of adding 2 quantities, which is the mathematical operation of addition. Write the word “sum” in the Addition column on your whiteboards.

Provide students time to write their responses on the whiteboards.

Teacher Note

Emphasize and model precise mathematical language—for example, “sum,” “difference,” “product,” and “quotient.”

We will list all of the words to make a reference sheet to use whenever you work with word problems. Tell me the words you think mean the same thing as indicated in the mathematical operations.

(addition: add, plus, sum, more than; subtraction: subtract, minus, difference, less than; multiplication: multiply, product, times, double, twice, triple; division: divide, quotient, halved, third, quartered; equal: is, is equal to, makes, totals, gives)

As students contribute, write a list for students to copy on to their *Demonstration Practice Sheet*.

Copy the list of words that we have generated at the top of your Demonstration Practice Sheet. You will use this list in today's lesson and in future lessons.

Provide students time to copy the list on the *Demonstration Practice Sheet*.

2. Guide students through defining variables to write an equation that represents the relationship in the contextual situation for problem 1 on the *Demonstration Practice Sheet*.

Have students complete the *Demonstration Practice Sheet* as you model the process of translating a verbal situation to an equation. Use different colored pencils or markers to help students identify the words that indicate variables and operations for the equation translation.

We will use 2 different colors. The first color will be used to identify the variables in the word problem and the second will be used to identify mathematical process words in the word problem. The mathematical process words will be the words that we brainstormed in our list today. These colors will be your variable color and process color for all word problems today.

Read problem 1 with students. Underline and have students underline the words “the sum of” on the *Demonstration Practice Sheet* with the process color.

The first step is to identify the information in the problem that indicates operations, numbers, and variables. This critical, or important, information will be included in our equation.

The problem begins with, “The sum of.” Underline the word “sum.” Based on our brainstorm list, what operation does the word “sum” indicate? (*addition*)

Write and have students write “+” above the words “the sum of.”

Write a plus sign above the word “sum.”

Pause for students to work.

Underline and have students underline the words “a number” using the variable color.

The next words in the problem are “a number.” Do we know what the number is? *(no)*

Recall the definition of a variable. *(a symbol, usually a letter, that represents a value or set of values)*

Because we don’t know what the value is, we will use a variable to represent it in our equation. Use the variable color to underline the words “a number.”

Remember we can choose any letter to represent this value. What letter would you like to use to represent this value?
(answers will vary; for this script, we will use the letter n)

Write and have students write “ n ” above the words “a number.”

We are going to use n to represent “a number.” Write n above “a number.”

Pause for students to work.

Underline and have students underline “7” and “is” with the process color.

The number 7 is next and should be underlined with the process color because we know the value that will be added to the variable n . The word “is” comes next. Underline the word “is.” Looking at the brainstorm list, what is the meaning of the word “is” in mathematics? *(equal, equal sign)*

Write and have students write “=” above the word “is.”

Write an equal sign above “is.”

Pause for students to work.

Underline and have students underline “15” with the process color.

Use your process color to underline the number “15.”

Pause for students to work.

Guide students through writing the equation for problem 1 using the underlined and translated words.

The next step in translating a word problem is to clearly define what the variables represent. On your Demonstration Practice Sheet, we will write down a description of the variable n . This will help us write the equation and help others to understand our work.

Write and have students write “ n = the value of the number” in the space provided next to “Variable(s)” on the *Demonstration Practice Sheet*.

Write “ n = the value of the number” to describe the variable.

The last step is to write the equation. We will use the variable, n , to write the equation.

Write and have students write “ $n + 7 = 15$ ” in the space provided next to “Equation” on the *Demonstration Practice Sheet*.

The sum of n and 7 is written as “ $n + 7$.” In mathematics, we use the word “is” to mean “is equal to.” So, we can write this equation as “ $n + 7 = 15$.”

Watch For



Students may try to write the equation in the order that they hear the words in the verbal statement. This may lead to problems or an incorrect equation. Go over the possible mistranslations and emphasize the relationship between the numbers and variables.

3. Guide students through defining variables to write an equation representing the relationship in the contextual situation for problem 2 on the *Demonstration Practice Sheet*.

Have students complete the *Demonstration Practice Sheet* as you model the process of translating a verbal situation into an equation. Continue

to use different colored pencils or markers to help students identify the process and variable words that will be used for the equation translation. Read problem 2 with students.

The first step of translating words into an equation is to identify the parts of the word problem that indicate operations, numbers, and variables.

Have students read the problem and determine the first word that they recognize from their brainstorm list.

Read problem 2 and look for the first word that you recognize from our brainstorm list.

What is the first word that you recognized? (*product*)

Looking at the brainstorm list we created earlier, what does the word “product” mean in mathematics? (*multiplication*)

Underline and have students underline the word “product” with the process color on the *Demonstration Practice Sheet*.

Write and have students write “*” above the word “product,” and have students underline the word product in the process color.

Because the word “product” means to multiply, write “*” above the word “product” and underline the word “product” in the process color.

Pause for students to work.

Underline and have students underline the words “first number” with the variable color. Then write and have students write the variable “*f*” above the words “first number.”

The next part of the word problem to underline is the number 3. The words “first number” follow. Again, we do not know what the first number is, so what should we use to represent it? (*a variable*)

Underline “first number” with the variable color. What variable do you want to use to represent “first number”?
(answers will vary; for this script we will use f)

Write “ f ” above “first number.”

Pause for students to work.

Underline and have students underline the word “is” and write “=” above the word.

Using our brainstorm list, what does the word “is” mean in mathematics? (equal, equal sign)

Using the process color, underline the word “is” and write the equal sign above it.

Pause for students to work.

Underline and have students underline the words “second number” and write the variable h above the words.

The last part of the word problem is “second number.” Do we know the value of the second number? (no)

If we do not know this value, what do we use to represent the second number? (variable)

Because the second number is different from the first number, we need to use a different variable to represent it. Let’s use h to represent the second number. Underline “second number” with your variable color and write “ h ” above it.

Define variables on the *Demonstration Practice Sheet*. Write and have students write “ f = the value of the first number” and “ h = the value of the second number” in the space provided.

The next step is to define variables for the unknown quantities. On your Demonstration Practice Sheet, write “ f = the value of the first number” and “ h = the value of the second number” on the Variables line.

Guide students through the process of writing the equation using the translated words and variables.

The third step is to put all of this information together into our equation. What operation does the word “product” indicate? (*multiplication*)

So the product of 3 and the first number means 3 times f . How do we write this mathematically? ($3f$)

The product of 3 and the first number, f , is equal to a second number, h . So, $3f$ is equal to h . How do we write this mathematically? ($3f = h$)

Using the variables that we have defined and the mathematical operations we have identified, the equation that describes the relationship is $3f = h$.

Write and have students write “ $3f = h$ ” in the space provided.

Write “ $3f = h$ ” in the space provided on your Demonstration Practice Sheet.

4. Continue with the translation and equation-writing process for problems 3 and 4.

Guide students through the thought process to define the variables and write the equation for each relationship. Use specific questions, such as the following, to elicit verbal responses from students and to check for their understanding.

- What is the first step for translating a verbal situation into an equation?
- What mathematical process does [insert word here] indicate?
- Does order matter in division? How do you know which value is divided by the other? Which value “comes first” or “goes on top”?
- What are the unknown quantities? How do you know?
- How do we write the equation using the translations and variables?

- How do you write [insert quantity] less than [insert quantity]?
 - How do you write [insert quantity] more than [insert quantity]?
5. Provide a summary of the content and process used in the lesson.

To review the key ideas of the lesson, ask students questions about translating word problems into mathematical equations.

In a word problem, what do we use to represent a value, or number, that we do not know? (*a variable*)

What are some examples of words that indicate addition? (*sum, altogether, more than*)

What are some examples of words that indicate subtraction? (*difference, less than, minus, take away*)

What are some examples of words that indicate multiplication? (*product, times, double, twice, triple, per, each*)

What are some examples of words that indicate division? (*quotient, halved, third, quartered, divided by*)

What are some examples of words that indicate equal? (*is, is equal to, makes, totals*)

Practice

Pair Practice

1. Have students work with a partner to answer the problems on the *Practice Sheet*.
2. Have student pairs define variables for each situation and then match the correct equation for the relationship. Tell students to be ready to justify their reasoning. Encourage students to use correct mathematical language.
3. Have student pairs present their reasoning by explaining their answers to the class. Encourage students to use correct mathematical language. Ask probing questions, such as the following, to elicit detailed responses:

- How did you know what the variable represented?
- What in the situation indicated that this equation represented the relationship?

Independent Practice

1. Have students define the variables and match the equation that best describes the relationship for each verbal situation on the *Independent Practice Sheet*.
2. Have students share their answers with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students explain why it is important to clearly define variables. Give students time to think of a response based on the examples that were provided.

- In a word problem, what do we use to represent a value, or number, that we do not know?
- What are some examples of words that indicate addition?
- What are some examples of words that indicate subtraction?
- What are some examples of words that indicate multiplication?
- What are some examples of words that indicate division?
- What are some examples of words that indicate equal?

Module 1: Variables**Lesson 8**

Lesson 8: Variables used in Verbal Translations, Part II

Lesson Objectives	<p>Students will define variables and translate verbal descriptions to multi-step equations from contextual situations.</p> <p>Students will use precise mathematical language when translating verbal descriptions to equations from contextual situations.</p>	
Vocabulary	No new words are introduced.	
Reviewed Vocabulary	difference, product, quotient, sum, variable	
Instructional Materials	Teacher	Student
	<ul style="list-style-type: none"> Teacher Masters (pp. 81-90) Overhead/document projector Colored pencils/markers 	<ul style="list-style-type: none"> Student Booklet (pp. 43-47) Colored pencils/markers Whiteboard with marker

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet* as part of Engage Prior/Informal Knowledge.

Engage Prior/Informal Knowledge

To open the lesson, lead a discussion to activate students' background knowledge and prerequisite skills of using variables to write generalizations and equations.

Discuss problem 1 on the *Cumulative Review Practice Sheet*.

The first problem on the Cumulative Review Practice Sheet asks you to find a generalization of the pattern you see in the table. The first step is to find the common difference. How do you find the common difference? (*answers may vary; $9 - 5 = 4$, $5 + 4 = 9$.*)

Look at each of the numerical statements in the table. What operations and numbers are the same for each statement? (*multiply by 4 and add 1*)

We always multiplied the term number by 4 and then added 1 to get the total.

Which generalization shows that for any number, n , we multiply n by 4, then add 1? ($4n + 1$)

Discuss problem 2 on the *Cumulative Review Practice Sheet*.

Look at problem 2. In this verbal situation, we are asked to select the equation that best represents the relationship.

What operation does the word “product” indicate? (*multiplication*)

What variables are used to represent the first number and second numbers? (*a and b*)

How do you write the product of a first number and 12 mathematically? ($12a$)

What symbol would you use to represent “is equal to” mathematically? ($=$)

Which equation best represents the relationship? ($12a = b$)

Preview

This lesson will build on students’ knowledge of the meaning and use of mathematical variables in contextual situations.

Today we will continue to discuss variables in contextual situations and write equations that represent the situations.

Demonstrate

Teacher Note

Emphasize and model precise mathematical language—for example, “sum,” “difference,” “product,” and “quotient.”

1. Guide students through defining variables to write an equation that represents the relationship in the contextual situation for problem 1 of the *Demonstration Practice Sheet*. Using an overhead projector or document projector, display the *Demonstration Practice Sheet* for students to see.

Ask students specific guiding questions to elicit think-alouds. Have students complete the *Demonstration Practice Sheet* as you guide them through the process of translating a verbal situation into an equation. Use different colored pencils or markers to help students identify the words that indicate variables and operations for the equation translation.

Like our previous work translating word problems into mathematical equations, we will use different colors to identify the variables and critical words in problem 1 on your Demonstration Practice Sheet. Select one color that will always be associated with variables.

Read problem 1 with students and have students identify the critical information.

Problem 1 on your Demonstration Practice Sheet says, “The second number is 5 more than twice the first number.” The first step is to identify the parts in the problem that indicate operations, numbers, and variables.

The problem begins with, “The second number.” Do we know what the value of the second number is? (*no*)

**Because we do not know the value, what do we use to represent this unknown value of the second number?
(*variables*)**

Underline and have students underline the words “second number” with the variable color on the *Demonstration Practice Sheet*.

Underline the words “second number” with the variable color.

Write and instruct students to write “*w*” above the words “second number.”

We need to select a variable to represent the value of the second number. It does not matter which letter we select, so I will select the variable, *w*, to represent the second number. Write the letter “*w*” above “second number.”

Pause for students to work.

Using our brainstorm list from last time, what is the next important word in the problem? (*is*)

What does “is” represent in mathematics? (*equal, is equal to, an equal sign*)

Underline and have students underline the word “is” with the process color and write “=” above the word.

Underline “is” with the process color and write “=” above.

Pause for students to work. Underline and have students underline “5” and “more than” with the process color on the *Demonstration Practice Sheet*.

The number 5 is next and should be underlined with the process color. What is the next important word(s) that we must underline in the problem? (*more than*)

What does “more than” mean in mathematics? (*addition*)

Teacher Note

If students have difficulty identifying the key words in the verbal situations, have them refer back to the Brainstorm section in Lesson 7.

Underline and have students underline “more than,” and write “+” above the words.

Underline “more than” and write “+” above it.

Pause for students to work.

The next word is “twice.” What does “twice” mean in mathematics? (*multiply by 2*)

Underline and have students underline the word “twice” with the process color and write “2*” above the word.

Underline “twice” and write “2*” above it.

Pause for students to work. Underline and have students underline “first number” with the variable color on the *Demonstration Practice Sheet*.

Lastly, we underline “first number.” Do we know what the value of the first number is? (*no*)

What color are we going to underline “first number” with? (*the variable color*)

Why? (*we do not know the value of the first number*)

We do not know the value of the first number, so we need to select a different variable to represent it.

Write and have students write “ x ” above the words “first number” on the *Demonstration Practice Sheet*.

Recall that it does not matter which letter we use, as long as it is different from the variable we used for the first number. Let’s use the variable, x , to represent the value of the first number. Underline “first number” and write “ x ” above it.

Pause for students to work. Guide students to write the equation for problem 1 using the underlined and translated words.

Based on our previous experience, what is the next step to translate the word problem into an equation? (*define the variables*)

On your Demonstration Practice Sheet, we will write down a description of the variables x and w . This will help us write the equation and help others to understand our work.

Write and have students write “ x = the value of the first number” and “ w = the value of the second number” in the space provided next to “Variable(s)” on your copy of the *Demonstration Practice Sheet*.

Write “ x = the value of the first number” and “ w = the value of the second number” to describe the variables.

Looking at how we have translated the parts of the word problem, the equation must begin with “ w =” to represent “The second number is.” How would you write “twice the first number”? ($2 \cdot x$; $2x$)

Remind students to switch the order when writing the equation when words like “more than” or “less than” are used in word problems.

Remember, when you see words like “more than” and “less than,” you are adding or subtracting the first quantity to/from the next quantity. So, to write equations from these words, the order of the quantities is switched. For example, in problem 1 it states “5 more than $2x$,” meaning 5 is added to the quantity $2x$. How should we write “5 more than $2x$ ” in the equation?
($2x + 5$)

This means that the equation for the word problem is $w = 2x + 5$.

Teacher Note

Students may write the equation as $w = 5 + 2x$, which is correct because of the commutative property of addition. Students should develop the habit of writing the equation in this order for subtraction problems.

Write and have students write “ $w = 2x + 5$ ” in the space provided on the *Demonstration Practice Sheet*.

Write “ $w = 2x + 5$ ” in the space provided on your *Demonstration Practice Sheet*.

Watch For



Students may try to write the equation in the order that they hear the words in the verbal statement. This may lead to problems or an incorrect equation. Go over the possible mistranslations and emphasize the relationship between the numbers and variables.

2. Guide students through defining variables to write an equation representing the relationship in the contextual situation for problem 2 on the *Demonstration Practice Sheet*.

Have students complete the *Demonstration Practice Sheet* as you model the process of translating a verbal situation to an equation. Use different colored pencils or markers to help students identify the words that indicate variables and operations for the equation translation. Ask specific questions to elicit verbal responses to check for their understanding.

Read problem 2 with students.

Look at problem 2 on the Demonstration Practice Sheet. This is another word problem that we need to translate to a mathematical equation. What is the first step in translating a word problem? (*underline and write the math symbols above the words*)

Have students read the problem and determine the first word that they recognize from their brainstorm list.

Read problem 2 to yourself and look for the first word that you recognize from our brainstorm list.

What is the first word that you recognized? (*double*)

What does the word “double” mean in mathematics? (*2*, multiplication by 2*)

Underline and have students underline the word “double” with the process color, and write “2*” above the word.

Using your process color, underline the word “double” and write “2*” above it.

Pause for students to work. Underline and write the variable “ f ” above the words “first number” with the variable color marker. Have students do the same.

In the word problem, what, specifically, is getting doubled? (*the first number*)

The words “first number” follow. Do we know what the first number is? (*no*)

What should we use to represent the first number? (*a variable*)

Underline “first number” using your variable color. What letter do you want to use to represent “first number”? (*answers will vary; for this script we will use f*)

Write “f” above the words “first number.”

Pause for students to work. Underline the words “is equal to” with the process color and write “=” above the words. Have students do the same.

What do the words “is equal to” indicate for a mathematical equation? (*equal, equal sign*)

Using the process color, underline the “is equal to” and write “=” above.

Pause for students to work. Have students continue to read the problem and look for the next word that indicates an operation.

Looking at the word problem for problem 2, what should be underlined next? (*difference*)

What does difference mean in mathematics? (*subtraction*)

Remember when you see difference, there will be two quantities listed to subtract.

Underline the word “difference” with the process color and write “–” above the word. Have students do the same.

Underline the word “difference” and write “–” above it.

Pause for students to work. Underline the words “second number” with the variable color and write “b” above the words. Have students do the same.

Next, the words “second number” should be underlined. Why do we underline these words? (*they represent the other unknown value*)

Let's use b to represent the value of "second number."
Underline with your variable marker and write b above.

Why do we need to use two different variables, f and b ? (*the values of the two numbers are different*)

On your *Demonstration Practice Sheet*, define the variable. Write and have students write " f = the value of the first number" and " b = the value of the second number" on the "Variable(s)" line.

The second step is to define variables for the unknown quantities. On your Demonstration Practice Sheet, how should the variable f be defined? In other words, what does the variable f represent? (*f = the value of the first number*)

Write " f = the value of the first number." How should the variable b be defined? (*b = the value of the second number*)

Write " b = the value of the second number."

Pause for students to work. Guide students through the process of writing the equation for problem 2, using the translated words and variables.

The third step is to put all of this information together into our equation. How do we write to double the first number, mathematically? (*$2f$*)

How do we write the difference of the second number and 1?
(*$b - 1$*)

Now we can put together the pieces. What is the equation that describes the relationship, "Double a first number is equal to the difference of a second number and 1?" (*$2f = b - 1$*)

Write and have students write " $2f = b - 1$ " on the *Demonstration Practice Sheet*.

Write " $2f = b - 1$ " in the space provided on your Demonstration Practice Sheet.

3. Continue with the translation and equation-writing process for problems 3 and 4.

Walk students through the thought process to define the variables and write the equation for each relationship. Use specific questions, such as the following, to elicit verbal responses from students and to check for their understanding.

- What is the first step in translating a verbal situation into an equation?
 - What mathematical operation does [insert word here] indicate?
 - What are the unknown quantities? How do you know?
 - How do you define the unknown quantities?
 - How do we write the equation using the translations and variables?
4. Provide a summary of the content and process used in the lesson.

To review the key ideas of the lesson, ask students questions about translating word problems into mathematical equations.

In a word problem, what do we use to represent a value, or number, that we do not know? (*a variable*)

What are some examples of words that indicate addition? (*sum, altogether, more than*)

What are some examples of words that indicate subtraction? (*difference, less than, minus, take away*)

What are some examples of words that indicate multiplication? (*product, times, double, twice, triple, per, each*)

What are some examples of words that indicate division? (*quotient, halved, third, quartered, divided by*)

What are some examples of words that indicate equal? (*is, is equal to, makes, totals*)

Practice

Pair Practice

1. Have students work with a partner to answer the problems on the *Practice Sheet*.
2. Have student pairs define variables for each situation and then match the correct equation for the relationship. Tell students to be ready to justify their reasoning. Encourage students to use correct mathematical language.
3. Have student pairs present their reasoning by explaining their answers to the class. Encourage students to use correct mathematical language. Ask probing questions, such as the following, to elicit detailed responses:
 - How did you know what the variable represented?
 - What in this situation indicated that this equation represented the relationship?

Error Correction Practice

1. Have students examine the 2 solution strategies presented and determine which strategy is incorrect. Have students justify why the solution strategy they chose is incorrect.
2. Have students share the reasoning for their answers to the class.

Independent Practice

1. Have students define the variables and match the equation that best describes the relationship for each verbal situation on the *Independent Practice Sheet*.
2. Have students share their answers with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Have students explain why it is important to clearly define variables. Give students time to think of a response based on the examples that were provided.

- In a word problem, what do we use to represent a value, or number, that we do not know?
- What are some examples of words that indicate addition?
- What are some examples of words that indicate subtraction?
- What are some examples of words that indicate multiplication?
- What are some examples of words that indicate division?
- What are some examples of words that indicate equal?

Module 1: Variables

Lesson 9

Lesson 9: Variables as Quantities That Vary: Independence and Dependence

Lesson Objectives	<p>Students will use variables to represent quantities that vary, including independent and dependent variables.</p> <p>Students will use precise vocabulary to communicate mathematical thinking coherently and clearly to peers and teachers.</p>	
Vocabulary	<p>quantities that vary: amounts that change in a mathematical relationship</p> <p>independent variable: the variable that determines the value of a second variable</p> <p>dependent variable: the variable whose value depends on the value of another variable</p>	
Reviewed Vocabulary	Variable	
Instructional Materials	Teacher	Student
	<ul style="list-style-type: none"> Teacher Master (pp. 91-100) Overhead/document projector Cups (2 colors) and counters 	<ul style="list-style-type: none"> Student Booklets (pp. 49-53) Whiteboard with marker

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet* as part of Engage Prior/Informal Knowledge.

Engage Prior/Informal Knowledge

To open the lesson, lead a discussion to activate students' background knowledge and prerequisite skills on translating word problems into equations. Have students refer to the Brainstorm section from Lesson 7 as necessary.

Discuss problem 1 on the *Cumulative Review Practice Sheet*.

Look at problem 1 from the Cumulative Review Practice Sheet. In this verbal situation, we are asked to select the equation that best represents the relationship.

What operation does the word sum indicate? (*addition*)

What variables are used to represent the value of the first number and the value of the second number? (*f and s*)

How do you write “the sum of the first number and 15,” mathematically? ($15 + f$)

What symbol would you use to represent “is equal to,” mathematically? ($=$)

Which equation best represents the relationship? ($15 + f = s$)

How did you know? (*answers will vary*)

Discuss problem 2 on the *Cumulative Review Practice Sheet*.

Look at problem 2. Again, we are asked to select the equation that best represents the relationship.

How do you write “the product of the first number and 6,” mathematically? ($6f$)

What symbol would you use to represent “is equal to,” mathematically? ($=$)

How do you write “the difference of the second number and 2,” mathematically? ($s - 2$)

Which equation best represents the relationship? ($6f = s - 2$)

How did you know? (*answers will vary*)

In each of these problems the variable has represented the value of a number. There are times that variables will represent actual things like the number of shoes or the total cost of buying pairs of shoes.

Preview

This lesson will build on students’ knowledge of the meaning and use of mathematical variables as quantities that vary.

Today we will use variables to represent *quantities that vary*, including *independent* and *dependent variables*.

Demonstrate

1. Introduce and define “quantities that vary.”

Using an overhead projector or document projector, display your *Variables* foldable for students to see. Point to the section on your *Variables* foldable and have students find it on their foldable. Write and have the students write the examples and the definition of “quantities that vary” in the section inside of the foldable.

Today we will use variables as *quantities that vary*. Recall the definition of variable is a symbol, usually a letter, that represents 1 value or a set of values. *Quantities that vary* are variables that represent a set of values or multiple values.

Look at your foldable and find the section labeled *quantities that vary*.

Watch For



Students may confuse “vary” with the meaning of “very.” Because the two words are homophones students may think “vary” means many or multiple. Be sure to clarify “vary” as meaning change before giving the definition of “quantities that vary.”

Have students explain the meaning of “quantities” and “vary.” Have students use the Think-Pair-Share routine with a partner to discuss the meanings.

Before we can discuss variables as *quantities that vary*, we need to make sure we understand the meaning of the words “quantity” and “vary.” In your own words, what does the word “quantity” mean? (*an amount of something*)

A quantity is an amount of something. An example of quantity is the amount of students in this class. Think of another example of a quantity, or amount, in the real world.

Pause for students to think. Have student pairs share their real world examples of an amount.

Pair with a neighbor to share your example of a *quantity*.

What does it mean to vary? (*change*)

One example of something that varies, or changes, is the amount of gas in a gas tank. As the car is driven, the amount of gas in the tank decreases or changes. Then the tank is filled up and again the amount changes. Think of another example of something that varies, or changes, in the real world with your partner.

Pause for students to think. Have student pairs share their real world examples of something that varies.

When we say quantities that vary, we mean amounts that change, or do not stay the same.

State and write the definition of “quantities that vary” on your hard copy of the *Variables* foldable. Have students write the definition in the definition section on their *Variables* foldable.

Today we will discuss variables that represent *quantities that vary*. *Quantities that vary* are amounts that change in a mathematical relationship.

Have students repeat the definition.

What is the definition of “*quantities that vary*?” (*amounts that change in a mathematical relationship*)

2. Present and discuss examples of variables as quantities that vary.

Write and have students write in the example section of the *Variables* foldable an example of variables as quantities that vary to help illustrate the definition.

An example of *quantities that vary* in the real world may help us understand the definition.

“Every day I eat pizza for lunch in the cafeteria. The cost of my lunch varies, or changes, depending on how many slices of pizza I buy.”

Write and have students write “ p = the number of slices of pizza I buy” and “ c = the total cost of my lunch” in the examples section of the *Variables* foldable.

Let the variable, p , represent the number of slices of pizza that I buy, and let the variable, c , represent the total cost of my lunch.

In the example section on your *Variables* foldable, write “ p = the number of slices of pizza I buy” and “ c = the total cost of my lunch.”

Notice that we used 2 different variables, p and c , in this example. We must use 2 different variables because there are 2 different quantities, or amounts, in our example.

One quantity is the number of slices of pizza that I buy, and another quantity is the total cost of my lunch.

Write “The variables, p and c , represent quantities that vary because they are amounts that change” on the *Variables* foldable.

Write “The variables, p and c , represent *quantities that vary* because they are amounts that change” on your Variables foldable. The cost of lunch varies, depending on the number of slices of pizza that I buy.

3. Define independent and dependent variables on the *Demonstration Practice Sheet*.

Use the example situation to introduce independent and dependent variables.

The 2 quantities in our example did not vary randomly. They varied together. There is a relationship between them. The number of slices of pizza had an effect on the cost of lunch. When 2 quantities vary together, we define 1 as the *independent variable* and 1 as the *dependent variable*.

Have students discuss with a partner their understanding of the word “independent” in real life situations.

In order to understand *independent variables*, we can discuss what it means to be independent in real life. Think about what it means to describe someone as independent.

Pause for students to think. Have student turn to a partner to discuss independence.

Discuss with a partner what it means to describe someone as independent.

Define the independent variable. Write and have students write the definition in the space provided on the *Demonstration Practice Sheet*.

The *independent variable* determines the value of the other variable. The value of an *independent variable* does not

depend on any other variable. Write this definition on the Demonstration Practice Sheet.

This is usually the value that is being manipulated or changed.

Have students repeat the definition.

What is the definition of *independent variable*? *(the variable that determines the value of the other variable; it does not depend on any other variable)*


Define the dependent variable. Write and have students write the definition in the space provided on the *Demonstration Practice Sheet*.

The value of the *dependent variable* depends on the value of the other variable. Write this definition on the Demonstration Practice Sheet.

This value is usually the result of the manipulation, or change, of the *independent variable*.

Have students repeat the definition.

What is the definition of *dependent variable*? *(the variable that depends on the value of the other variable)*

 <p>Watch For</p>	<p>Students may have difficulty with the concept of independent and dependent variables. Provide real life situations that they can connect to the abstract definition. For example, discuss how the amount of money a person makes depends on the number of hours that they work.</p>
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4. Use the real-world situation in problem 1 on the *Demonstration Practice Sheet* to reinforce independent and dependent variables.

Look at problem 1 and recall the situation that we discussed earlier. The amount that I pay for lunch each day varies based on the number of slices of pizza that I buy.

Since we know that the number of pizza slices purchased and the amount of money paid are the *quantities that vary*, we need to identify the *independent* and *dependent variable*.

Determine the independent and dependent variable for the 2 quantities that vary in the real-world situation. Ask students to think about the question of dependence, i.e., “What depends on what?”

We must figure out which quantity depends on the other quantity. To figure this out, I ask myself, does the amount of pizza slices depend on the cost? Or, does the cost depend on the number of pizza slices purchased?

Underline and have students underline “total cost,” “based on,” and “number of pizza slices.”

When I read the situation again, I will underline words that may give me a hint about which quantity depends on which.

One *quantity that varies* is the total cost, so I will underline the words “total cost.” Another *quantity that varies* is the number of pizza slices, so I will underline the words “number of pizza slices.” The situation says that the cost of lunch is based on the number of pizza slices. The words “based on” are key words, so I will underline them. The cost of lunch is based on the number of slices of pizza.

Write and have students write, “cost of lunch depends on the number of pizza slices purchased” in the blanks provided for problem 1.

I choose how many pizza slices I want to buy each day and then I am charged based on how much pizza I bought. Because the cost of lunch depends on the number of pizza slices purchased, c is the *dependent variable* and p is the *independent variable*.

Write “cost of lunch” in the first blank and “the number of pizza slices purchased” in the last blank provided for problem 1. This means we would say, “the cost of lunch depends on the number of pizza slices purchased.”

Teacher Note

Real-world examples can be very difficult for students to identify as independent and dependent quantities. Be careful of the wording because the same problems can be stated differently, resulting in different independent and dependent quantities.

Write and have students write, “ p = the number of slices of pizza” for independent variable, and “ c = total cost of lunch” for dependent variable.

Looking at your Demonstration Practice Sheet, what should we write in the blank for *independent variable* and *dependent variable*? (p = the number of slices of pizza and c = total cost of lunch)

Write “ p = the number of slices of pizza” for the *independent variable* and “ c = total cost of lunch” for the *dependent variable*.

5. Determine the independent and dependent variable for the 2 quantities that vary in the real-world situation in problem 2 on the *Demonstration Practice Sheet*.

Look at problem 2. “Sandra has a job passing out flyers for a sandwich shop. The total number of flyers that she passes out each day changes based on the number of people that walk by her that day.”

The first step is to identify the *quantities that vary*. What are the 2 *quantities that vary*? (the number of people that walk by and the total number of flyers passed out)

Now we will use variables to represent each *quantity that varies*. What variable would you like to use for the number of people that walk by? (answers will vary, this script uses p)

What variable would you like to use to represent the total number of flyers passed out? (answers will vary, this script uses f)

Write and have students write the variables chosen in the spaces provided on the *Demonstration Practice Sheet*.

Write “*p*” in the blank for number of people that walk by and “*f*” in the blank for total number of flyers passed out.

Underline and have students underline key words in the word problem for problem 2 on the *Demonstration Practice Sheet*.

Look at the situation and underline both of the *quantities that vary* and any other words that may give a hint about which quantity depends on which.

Pause for students to work. When students are finished underlining, check for their understanding. Students should have underlined the words “the number of flyers,” “determined by,” and “the number of people.”

Teacher Note

If students have trouble finding key words, stop and explicitly teach the material.

Since we know the *quantities that vary*, we need to identify which are the *independent* and *dependent variables*.

We should have underlined the words “the number of flyers,” “determined by,” and “the number of people.”

Give students time to think about both cases. Do not require an answer; instead generate a discussion.

To figure this out, we ask ourselves, “Does the number of people that walk by depend on the number of flyers passed out? Or, does the number of flyers passed out depend on the number of people that walk by?”

Let’s discuss a possible situation to help us decide. Suppose you have a large stack of flyers to pass out. If 5 people walk by, how many fliers can you pass out? (5)

Why only 5? (*there were only 5 people that walked by*)

Which event had to happen first, people walking by or passing out flyers? (*people walking by*)

Back to our original problem, which quantity depends on the other quantity? (*the number of flyers passed out depends on the number of people that walk by*)

The words “determined by” are another way to say depends on.

Write and have students write, “the number of flyers passed out depends on the number of people that walk by” in the blanks provided for problem 2 on the *Demonstration Practice Sheet*.

How do we think we should complete the “depends on” blanks on the Demonstration Practice Sheet? (*number of flyers passed out depends on the number of people that walk by*)

Because the number of flyers passed out depend on the number of people that walk by, f is the *dependent variable* and p is the *independent variable*.

Write and have students write, “ p = the number of people that walk by” for independent variable and “ f = the number of flyers passed out” for dependent variable.

What do we write in the *independent variable* blank? (p = the number of people that walk by)

What do we write in the *dependent variable* blank? (f = the number of flyers passed out)

Write “ p = the number of people that walk by” in the *independent variable* blank and “ f = the number of flyers passed out” in the *dependent variable* blank.

6. Continue with the translation and independent/dependent variable process for problem 3.

Use specific questions, such as the following, to elicit verbal responses from students and to check for understanding.

- What are the 2 quantities that vary?
- Which quantity depends on which? How do you know?
- Does the total amount of money made depend on the number of cars washed?
- Does the number of cars washed depend on the total amount of money made?
- Which is the independent variable? How do you know?

Practice

Pair Practice

1. Have students work with a partner to answer the problems on the *Practice Sheet*.
2. Have student pairs read each real-world situation and determine the independent and dependent variables.
3. Have student pairs present their reasoning by explaining their answers to the class. Encourage students to use correct mathematical language. Ask probing questions to elicit detailed responses, including the following:
 - What are the 2 quantities that vary?
 - Which quantity depends on which?
 - Does the first quantity depend on the second quantity?
 - Does the second quantity depend on the first quantity?
 - Which is the independent variable? How do you know?

Independent Practice

1. For problems 1–2, have students select the equation that best describes the corresponding set of numerical equations.
2. Have students share their answers with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total number correct at the top of the page.

Closure

Review the key ideas. Give students time to think of a response to the following, based on the examples from the lesson.

- We have written definitions and worked through examples. How would you explain, in your own words, variables as *quantities that vary*?
- In your own words, how would you explain *independent variables*?
- In your own words, how would you explain *dependent variables*?

Module 1: Variables

Lesson 10

Lesson 10: Variables as Quantities That Vary - In Context, Part I

Lesson Objectives	<p>Students will use variables to translate verbal descriptions into equations representing contextual situations.</p> <p>Students will create and use representations to organize, record, and translate contextual situations into equations.</p>	
Vocabulary	unit rate: a ratio of 2 quantities in which the second quantity in the comparison is 1.	
Reviewed Vocabulary	dependent, independent, quantities that vary, variable	
Instructional Materials	Teacher	Student
	<ul style="list-style-type: none"> Teacher Masters (pp. 101-112) Overhead/document projector Colored pencils/markers 	<ul style="list-style-type: none"> Student Booklet (pp. 55-60) Colored pencils/markers

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet* as part of Engage Prior/Informal Knowledge.

Engage Prior/Informal Knowledge

To open the lesson, lead a discussion to activate students' background knowledge and prerequisite skills on translating word problems into equations. Have students refer to the Brainstorm section from Lesson 7 as necessary.

Discuss problem 1 on the *Cumulative Review Practice Sheet*.

Look at problem 1 from the Cumulative Review Practice Sheet. In this verbal situation, we are asked to select the equation that best represents the relationship.

How do you write “the product of the first number and 6,” mathematically? ($6f$)

What symbol would you use to represent “is equal to,” mathematically? ($=$)

What operation does the word “quotient” indicate? (*division*)

How do you write “the quotient of twenty-four and a second number,” mathematically? ($24 \div s$)

Which equation best represents the relationship? ($6f = 24 \div s$)

Discuss problem 2 on the *Cumulative Review Practice Sheet*.

Look at problem 2. In this verbal situation, we are asked to label each quantity that varies as independent or dependent.

What are the 2 quantities that vary? (*the number of miles that Aldo drives each month and total amount of money spent on gas*)

What key words do you see in the word problem that may give a hint about which quantity depends on which? (*money, is determined by, and miles*)

Does the number of miles driven each month depend on the amount of money spent on gas, or does the total amount of money spent on gas depend on the number of miles driven?
(the amount of money spent depends on miles driven each month)

Which is the independent variable? (m , the number of miles driven each month)

Which is the dependent variable? (t , the total amount of money spent on gas)

Preview

This lesson will build on students' knowledge of variables in contextual situations. Students will define and use variables as quantities that vary to create equations to represent contextual situations.

Today we will continue working with contextual situations, or word problems. We will define variables and what they represent in situations. Then, we will create equations to represent the situations.

Demonstrate

1. Guide students through defining the variables and writing the equation for the contextual situation in problem 1 on the *Demonstration Practice Sheet*.

Have students complete the *Demonstration Practice Sheet* as you model the process of translating a verbal situation into an equation. Using an overhead projector or document projector, display your *Demonstration Practice Sheet* for students to see and copy as you write. Remind students that it may be helpful to use different colors to help identify the variables and critical information.

Read problem 1 from the Word Problem section with students and have them underline the critical information.

Follow along as I read the word problem for problem 1. In the Word Problem section, we will underline the critical


information. Recall that the critical information is the important information that we will use to write an equation. “Alicia works at an electronic store, where she makes \$11 each hour. Find the total amount of money Alicia makes.”

Underline and have students underline “\$11 each hour” on the *Demonstration Practice Sheet*.

Underline “\$11 each hour” in the Word Problem section for problem 1.

In mathematics, the word “each” indicates a relationship between 2 quantities that vary.

Identify the quantities that vary in the word problem for problem 1. Determine the variables and describe what they represent. Write and have students write the variables and the description on the *Demonstration Practice Sheet* in the Define Variables section.

 <p>Watch For</p>	<p>Students may assume that some variables are letters that are used as abbreviations. For example, they may choose the variable “C” to represent Charlie instead of the value of Charlie’s age in years. When defining variables in contextual situations, emphasize that the variable stands for the value or quantity.</p>
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Now look at the Define Variable(s) section for problem 1. We need to define the 2 quantities that vary. One of the quantities that vary is the amount of money that Alicia makes.

Remember, when you choose letters to represent values for a situation, it is helpful to use letters that relate to the meaning. What variable should we use to represent the amount of money Alicia makes? *(answers will vary, but this script uses t for the total money made)*

Write and have students write “ t ” in the first blank and “the total amount of money Alicia makes” in the second blank in the Define Variable(s) section.

We will use the variable t to represent the total amount of money Alicia makes. Write “ t ” and “the total amount of money Alicia makes” in the space provided in the Define Variable(s) section.

Pause for students to work.

The other quantity that varies is the number of hours Alicia works. What variable should we use to represent the number of hours worked? *(answers will vary, but this script uses h for the number of hours worked)*

Write and have students write “ h ” in the first blank and “the total number of hours Alicia works” in the second blank in the Define Variable(s) section.

We will use the variable h to represent the total number of hours Alicia works. Write “ h ” and “number of hours Alicia works” in the space provided in the Define Variable(s) section.

Pause for students to work. Discuss with students how to relate the dependent and independent variables for problem 1.

Recall that independent variables determine the value of other variables.

The amount of money made changes, depending on the number of hours worked. So, t , the amount of total money made, depends on h , the number of hours worked. The variable t is the dependent variable because it depends on h . So, the variable h is the independent variable.

Label and have students label each variable with an I or D to indicate independent and dependent variable.

Next to the variable h , write “I” to indicate independent variable. Write “D” next to the variable t to indicate the dependent variable.

Guide students through the process of generating the equation that best represents the situation in problem 1 in the Write an Equation section of the *Demonstration Practice Sheet*.

In the Write an Equation section, we will write an equation to determine the amount of money Alicia makes for any number of hours she works.

Use the translation from the Word Problem section in problem 1 to write an equation in the Write an Equation section of the *Demonstration Practice Sheet*.

We will use the variables h and t to write the equation. The variable t represents the total amount of money, so we will write “ t ” in the “Total money made” box.

The \$11 dollars each hour is a *unit rate* describing the relationship between the 2 quantities.

Define “unit rate” for students as a way to direct the mathematical thinking process.

***Unit rate* is a ratio of 2 quantities in which the second quantity in the comparison is 1.**

So, if Alicia works 1 hour, she makes \$11 because her rate of earning is \$11 each hour. What does she make if she works 2 hours? (\$22)

How do you know? *(for each hour she earns \$11; multiply 11 by the number of hours)*

If we multiply 11 by the number of hours to calculate Alicia’s total money, how do we write it mathematically? ($11h$)

Write and have students write “ $11h$ ” in the box labeled “Calculate the money earned” in the Write an Equation section.

This means that in the box labeled “Calculate the money earned,” we write “ $11h$.”

Point to the boxes as you say the equation in the Write an Equation section of the *Demonstration Practice Sheet*.

The equation that represents this relationship is $11h = t$. Write this equation under the boxes.

2. Guide students through defining variables and writing the equation for the contextual situation in problem 2 on the *Demonstration Practice Sheet*.

Read problem 2 with students. Have students complete the *Demonstration Practice Sheet* as you model the process of translating a verbal situation into an equation. It will be helpful for you and the students to use different colors to identify the variables and critical information.

Follow along as I read the word problem for problem 2. In the Word Problem section, we will underline the critical information.

"The total cost of shipping a package is \$2 per pound. Find the total cost of shipping a package."

Underline and have students underline “\$2 per pound” on the *Demonstration Practice Sheet*.

Underline “\$2 per pound.”

In mathematics, the word “per” also indicates a relationship between 2 quantities that vary. The statement “\$2 per pound” is a *unit rate*. Which do you think are the 2 quantities in this relationship? (*answers will vary*)

Teacher Note

If students have difficulty identifying the key words in the verbal situations, have them refer back to the Brainstorm section in Lesson 7.

Identify the quantities that vary in problem 2. Determine the variables and describe what they represent. Write and have students write the variables and the description in the Define Variable(s) section of the *Demonstration Practice Sheet*.

In the Define Variables section, we will define the 2 quantities that vary. One of the quantities that vary is the total cost of the package.

What variable should we use to represent the total cost of shipping the package? *(answers will vary, but this script uses t for the total cost of shipping the package)*

Write and have students write “ t ” in the first blank and “total cost of shipping the package” in the second blank of the Define Variable(s) section in problem 2.

We will use the variable t to represent the total cost of shipping the package. Write “ t ” and “total cost of shipping the package” in the spaces provided in the Define Variable(s) section.

Pause for students to work.

What is the other quantity that varies? *(the weight of the package in pounds)*

What variable should we use to represent the weight of the package in pounds? *(answers will vary, but this script uses w for the weight of the package in pounds)*

Write and have students write “ w ” in the first blank and “weight of the package in pounds” in the second blank of the Define Variable(s) section in problem 2.

We will use the variable w to represent the weight of the package in pounds. Write “ w ” and “weight of the package in pounds” in the space provided in the Define Variable(s) section.

Pause for students to work. Have students think about which variable is independent and which is dependent for the 2 quantities that vary in problem 2. Have students Think-Pair-Share to begin the discussion.

Think about which variable is the independent and which is the dependent variable. Does the total cost depend on the weight, or does the weight depend on the total cost?

Pause for students to think. Have them share their reasoning with a neighbor.

Pair with your neighbor to share which is the dependent variable and which is the independent variable.

Pause for students to discuss.

Have student pairs share what they discussed.

Teacher Note

Use a method to promote active student response and encourage all students to participate in class discussion. For example, randomly draw popsicle sticks or cards with student's names on them when selecting students to share their answers or reasoning.

Which variable is the independent variable and which is the dependent variable? (w is independent and t is dependent)

How do you know? (answers will vary; the total cost of shipping the package depends on the weight of the package)

Label and have students label each variable with an “I” or “D” to indicate independent and dependent variables.

Next to the variable w , write “I” to indicate independent variable. Write “D” next to the variable t to indicate the dependent variable.

Guide students through the process of generating the equation that best represents the situation in problem 2 in the Write an Equation section of the *Demonstration Practice Sheet*.

Using the Write an Equation section, we will write an equation to find the cost of shipping a package for any weight of the package.

Write and instruct students to write “ t ” in the box labeled “Total cost” in the Write an Equation section.

We will use the variables w and t to write the equation. The variable t represents the total cost of shipping the package, so we will write “ t ” in the “Total cost” box.

Pause for students to work.

What variable did we use to represent the package weight, in pounds? (w)

Pause for students to work.

Write and instruct students to write “ w ” in the box labeled “Calculate the package shipping cost.”

How can we use the *unit rate*, \$2 per pound, to calculate the shipping cost? (*multiply the weight, w , by 2*)

How do we write this mathematically? ($2w$)

Write “ $2w$ ” in the box labeled “Calculate the package shipping cost.”

Point to each of the boxes, in the Write an Equation section, as you say the equation that represents the relationship in problem 2.

The equation that represents this relationship is $2w = t$. Write this equation under the boxes.

3. Repeat the process outlined above for defining and describing the variables, and writing the equation for problem 3.

Guide students through the thought process, using specific questions such as the following, to elicit verbal responses from students and check for understanding:

- What is the first step when working with word problems?
- What is the unit rate?
- How should we describe the variable?
- Which variable is independent? Which variable is dependent? How do you know?
- What does [insert word from problem] mean in mathematics?
- How do you write the equation that represents this situation?

4. Provide a summary of the content and process used in the lesson.

To review the key ideas of the lesson, ask students questions about the quantities that vary, the independent variable, and the dependent variable.

From the word problems that we read today, what is an example of a quantity that varies? (*answers will vary*)

What does the *unit rate* tell you? (*the comparison/relationship between the quantities that vary*)

Give examples of an independent variable and a dependent variable? (*answers will vary*)

How can you tell which quantity depended on which? (*answers will vary*)

Practice

Pair Practice

1. Have students work with a partner to answer the problems on the *Practice Sheet*.
2. Have student pairs define variables and select the equation that best represents the situation. Tell students to be ready to justify their reasoning. Encourage students to use correct mathematical language.
3. Have student pairs present their reasoning by explaining their answers to the class. Encourage students to use correct mathematical language. Ask probing questions, such as the following, to elicit detailed responses:
 - What are the unknown quantities? How do you know?
 - What is the unit rate?
 - Which variable is independent and which is dependent? How do you know?
 - How do you know that this equation represents this situation?

Error Correction Practice

1. Have students examine the 2 solution strategies presented and determine which strategy is incorrect. Have students justify why the solution strategy that they chose is incorrect.
2. Have students share the reasoning for their answers with the class.

Independent Practice

1. Have students match each contextual situation on the *Independent Practice Sheet* to the equation that best describes it.
2. Have students share their answers with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total correct at the top of the page.

Closure

Review the key ideas. Have students explain independent and dependent variables. Give students time to think of a response to the following, based on the examples from the lesson.

- Give a real-life example of 2 quantities that vary.
- Which of these is the independent variable?
- Which of these is the dependent variable?

Module 1: Variables**Lesson 11**

Lesson 11: Variables as Quantities That Vary – In Context, Part II

Lesson Objectives	<p>Students will use variables to translate verbal descriptions into equations and tables representing contextual situations.</p> <p>Students will create and use representations to organize, record, and translate contextual situations into equations.</p>	
Vocabulary	No new words are introduced.	
Reviewed Vocabulary	constant term, dependent, independent, input, output, quantities that vary, unit rate, variable	
Instructional Materials	Teacher	Student
	<ul style="list-style-type: none"> Teacher Masters (pp. 113-126) Overhead/document projector Colored pencils/markers 	<ul style="list-style-type: none"> Student Booklet (pp. 61-67) Colored pencils/markers

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet* as part of Engage Prior/Informal Knowledge.

Engage Prior/Informal Knowledge

To open the lesson, lead a discussion to activate students' background knowledge and prerequisite skills about independent and dependent variables and translating word problems into equations.

Discuss problem 1 on the *Cumulative Review Practice Sheet*.

Problem 1 on the Cumulative Review Practice Sheet asks you to label each quantity that varies as independent or dependent.

Which quantity is the independent quantity? (*the number of people attending*)

How do you know? (*the problem says that the number of people attending determine the cost; a number of people have to attend to have a cost*)

Therefore, the cost of the dinner party is dependent.

Discuss problem 2 on the *Cumulative Review Practice Sheet*.

Problem 2 on the Cumulative Review Practice Sheet asks you to select the equation that represents the relationship in the word problem.

What are the two quantities that vary? (*number of pounds and total cost*)

What are the variables that are used? (*p and t*)

Which equation represents the relationship in the word problem? (*A*)

How do you know? (*it costs \$5 per pound*)

Remember that “\$5 per pound” is the unit rate comparing the quantities that vary.

Preview

This lesson will build on students’ knowledge of variables in contextual situations. Students will define and use variables as quantities that vary to write equations and create tables to represent contextual situations.

Today we will continue working with contextual situations, or word problems. We will define variables and what they represent in situations. Then, we will create equations and tables to represent the situations.

Demonstrate

1. Guide students through defining the variables, generating the table, and writing the equation for the contextual situation in problem 1 on the *Demonstration Practice Sheet*.

Have students complete the *Demonstration Practice Sheet* as you model the process of translating a verbal situation into an equation and a table to represent the word problem. Using an overhead projector or document projector, display your *Demonstration Practice Sheet* for students to see and copy as you write. Remind students that it may be helpful to use different colors to help identity the variables and critical information.

Read problem 1 to students from the Word Problem section and have students underline the critical information.

Follow along as I read the word problem for problem 1. In the Word Problem section, we will underline the critical, or important, information. “Midori is saving up to buy a car. Her account has an initial balance of \$300 and she is depositing \$40 per month into her account. Find an equation to represent the amount Midori has saved.” What should we underline as critical information? (initial balance of \$300, \$40 per month)

Underline and have students underline “initial balance of \$300” and “\$40 per month” on the *Demonstration Practice Sheet*.

Underline “initial balance of \$300” and “\$40 per month” in the Word Problem section for problem 1.

Pause for students to work. Write and have students write “+ 300” above the words “initial balance of \$300.”

When we see the words “initial balance” in word problems, this is a beginning amount and should be added. This value is a constant amount that does not change value. Write “+ 300” above the words “initial balance of \$300.”

Have students discuss with a partner what are the 2 quantities that vary in problem 1.

Recall that previously we discussed the word “per” to indicate a relationship between 2 quantities that vary.

What is the unit rate in this relationship? (*\$40 per month*)

Turn to a neighbor and discuss what you think are the 2 quantities that vary.

Pause for students to think and discuss.

What do you and your neighbor think are the 2 quantities that vary? (*answers may vary; money saved and months*)

Identify the quantities that vary in the word problem for problem 1. Determine the variables and describe what they represent. Write and have students write the variable and its meaning in the Define Variable(s) section on the *Demonstration Practice Sheet*.

Watch For



Students may assume that some variables are letters that are used as abbreviations. For example, they may choose the variable “c” to represent Charlie instead of the value of Charlie’s age in years. When defining variables in contextual situations, emphasize that the variable stands for the value or quantity.

In the Define Variable(s) section for problem 1, we will define the 2 quantities that vary and represent each quantity with a variable. One of the quantities that vary is the number of months that Midori is saving.

Remember, when we choose letters to represent values for a situation, it is helpful to use letters that remind us of what they represent. What variable should we use to represent the number of months? *(answers will vary, but this script uses m for the number of months)*

Write and have students write “ m ” in the first blank in the Define Variable(s) section for problem 1. Write the meaning of the variable, “the number of months,” in the second blank.

We will use the variable m to represent the number of months that Midori saves. Write “ m ” and “the number of months” in the space provided in the Define Variable(s) section on the Demonstration Practice Sheet.

Pause for students to work.

The other quantity that varies is the total amount of money saved in Midori’s account. What variable should we use to represent the total amount of money saved? *(answers will vary, but this script uses t for the total amount of money)*

Write and have students write “ t ” in the first blank in the Define Variable(s) section for problem 1. Write the meaning of the variable, “the total amount of money saved,” in the second blank.

We will use the variable t to represent the total amount of money Midori saved. Write, “ t ” and “the total amount of money saved” in the space provided in the Define Variable(s) section.

Pause for students to work. Determine the independent and dependent variables for the 2 quantities that vary in problem 1.

Recall that the independent variable determines the value of the dependent variable.

The total amount of money saved changes, depending on the number of months. So, t , the total amount of money saved, depends on m , the number of months. The variable t is the dependent variable because it depends on m . So, the variable m is the independent variable.

Label and have students label each variable with “I” or “D” to indicate the independent and dependent variable.

Next to the variable m write “I” to indicate the independent variable. Write “D” next to the variable t to indicate the dependent variable.

Guide students through generating the equation that best represents the situation in problem 1 in the Write an Equation section of the *Demonstration Practice Sheet*.

In the Write an Equation section, we will write an equation to determine the amount of money Midori saves for any number of months.

Use the translation from the Word Problem section for problem 1 to write an equation in the Write an Equation section. Point to each of the boxes in the Write an Equation section as you fill them in on the *Demonstration Practice Sheet*. Write and have students write “ t ” in the box labeled “Total Saved.”

We will use the variables m and t to write the equation. The variable, t , represents the total amount of money saved, so we will write “ t ” in the box labeled “Total Saved.”

How do we use the unit rate “\$40 per month” to calculate the amount of money Midori deposits? (*multiply 40 by the number of months she saves*)

If Midori saved for 3 months, how much money did she save in that time? (\$120)

How do we represent 40 times the number of months, m , mathematically? ($40m$)

Write and have students write “ $40m$ ” in the box labeled “Amount Deposited” and “300” in the box labeled “Initial Balance” in the Write an Equation section on the *Demonstration Practice Sheet*.

The product of 40 and m is the amount deposited. Write “ $40m$ ” in the box labeled “Amount Deposited.” The second box is labeled “Initial Balance.” What was the initial balance that Midori started with? What should we write in the Initial Balance box? (\$300)

Why does the \$300 not have a variable? (*the initial balance is a set amount and does not change, it is a constant term*)

The initial balance of \$300 is a constant term in the equation. Recall that a constant term is a term that contains no variables and does not change.

The equation that represents this relationship is $40m + 300 = t$. Write this equation under the boxes.

Fill in and have students fill in labels for the columns in the table with the independent variable in the first column and the dependent variable in the last column in the Make a Table section of the *Demonstration Practice Sheet*.

Now, in the Make a Table section, we need to label the columns before we can create a table of this relationship. The independent variable goes into the first column in the table. What is the independent variable? (m)

How do you know? *(answers will vary; the total amount of money saved depends on the number of months)*

Write and have students write the independent variable, m , in the first column of the table in the Make a Table section.

In the first column, write “ m ” for the number of months.

We will write the dependent variable in the last column. What is the dependent variable? *(t)*

Write the dependent variable, t , in the second column of the table in the Make a Table section and instruct students to do the same.

In the last column, write “ t ” for the total amount of money saved.

Guide students through filling out the Process column by giving values for the independent variable. Write and have students write in the Process column the mathematical operations to result in the dependent column value. Connections to the equation and what is written in the Process column should be made explicit to students.

The Process column helps us determine how much money Midori has saved, based on the number of months she saved. We will use this column to keep track of our calculations.

Write and have students write “2” in the column for m and “ $40(2) + 300$ ” in the Process column.

Write “2” in the column for m . If Midori has saved for 2 months, how do we figure out how much money she has in her account? *(we start with 300 and then add $40(2)$)*

What is the output or total amount she has saved for 2 months? *(\$380)*

Write and have students write “380” in the column for t .

Write “380” in the column for t .

Because Midori saved for 2 months and each month she deposits \$40, we multiplied 40 by 2 to get 80. Since her account already had \$300, we add 80. Write “ $40(2) + 300$ ” in the Process column.

For the input value of 2 months, the output value was \$380 dollars saved.

Does the Process column match the equation we wrote in the Write an Equation section? (yes)

Teacher Note

Allow students to use a calculator for computations as necessary. It is important that the students understand the concept and do not get lost in the calculations.

Continue this process by having students supply input values for the number of months, calculate the output or total amount of money saved, and record the calculations in the Process column on the *Demonstration Practice Sheet*.

2. Guide students through defining variables, writing the equation, and generating the table for the contextual situation in problem 2 on the *Demonstration Practice Sheet*.

Read problem 2 to students. Write and have students write as you translate a verbal situation into an equation. Underline and have students underline the critical information in the Word Problem section of problem 2.

Follow along as I read the word problem for problem 2. In the Word Problem section, we will underline the critical information.

“The cost of a taxi ride in Chicago is \$0.50 per mile plus an additional fee of \$5. Find the total cost of a taxi ride.” What is the critical information that we underline in this problem?
(*\$0.50 per mile, plus an additional fee of \$5*)

Underline and have students underline “\$0.50 per mile” and “plus an additional fee of \$5” on the *Demonstration Practice Sheet*.

Underline “\$0.50 per mile” and “plus an additional fee of \$5” in the Word Problem section for problem 2.

Pause for students to work.

What does “\$0.50 per mile” represent in this problem? (*unit rate, relationship between the quantities that vary*)

How do we use the unit rate “\$0.50 per mile” in this problem? (*multiply the number of miles by 0.50*)

How do you translate “plus an additional fee of \$5”? (+ 5)

Write and have students write “+ 5” above the words “plus an additional fee of \$5.”

Write “+ 5” above the words “plus an additional fee of \$5.”

Pause for students to work.

An additional fee means that no matter how far you ride in the taxi, \$5 is added to the cost per mile.

Identify the quantities that vary. Determine the variables and describe what they represent. Write and have students write the variables and the description in the Define Variables section on the *Demonstration Practice Sheet*.

In the Define Variables section, we will define the 2 quantities that vary and use variables to represent each quantity. One quantity that varies is the total cost of the taxi ride.

What variable should we use to represent the total cost of the taxi ride? (*answers will vary, but this script uses t for the total cost of the taxi ride*)

Write and have students write “ t ” in the first blank and the meaning, “the total cost of the taxi ride,” in the second blank in the Define Variable(s) section for problem 2.

We will use the variable t to represent the total cost of the taxi ride. Write “ t ” and “the total cost of the taxi ride” in the space provided in the Define Variable(s) section.

Pause for students to work.

What is the other quantity that varies? (*number of miles driven*)

What variable should we use to represent the number of miles driven? (*answers will vary, but this script uses m for the number of miles driven*)

Write and have students write “ m ” in the first blank and the meaning, “the number of miles driven,” in the second blank in the Define Variable(s) section for problem 2.

We will use the variable m to represent the number of miles driven. Write “ m ” and “number of miles driven” in the space provided in the Define Variable(s) section.

Pause for students to work. Have students discuss with a partner the independent and dependent variable in problem 2. Have students Think-Pair-Share to begin the discussion.

Think about which variable is independent and which is dependent. Does the total cost depend on the number of miles, or does the number of miles depend on the total cost?

Pause for students to think. Have students share their reasoning with a neighbor.

Pair with your neighbor to share which variable is independent and which is dependent.

Pause for student to share. Have student pairs share what they discussed.

Which variable is the independent variable and which is the dependent variable? (*m is independent and t is dependent*)

How do you know? (*answers will vary; the total cost of the taxi ride depends on the number of miles driven*)

Label and have students label each variable with “I” or “D” to indicate the independent and dependent variable.

Next to the variable m write “I” to indicate the independent variable. Write “D” next to the variable t to indicate the dependent variable.

Guide students through generating the equation that best represents the situation in problem 2. Write the equation in the Write an Equation section of the *Demonstration Practice Sheet*.

Using the Write an Equation section, we will write an equation to find the cost of a taxi ride for any number of miles driven.

We will use the variables m and t to write the equation. The variable t represents the total cost of the taxi ride, so we will write “ t ” in the first box.

Pause for students to work.

What variable did we use to represent the number of miles? (m)

How do we calculate the cost of the miles driven? (*multiply the number of miles by 0.50*)

Write and have students write “ $0.50m$ ” in the box labeled “Cost of Miles” in the Write an Equation section.

We calculate the cost of the miles driven by multiplying the number of miles by 0.50. What should we write in the box labeled “Cost of Miles?” ($0.50m$)

Write “ $0.50m$ ” in the box labeled “Cost of Miles” in the Write an Equation section on your Demonstration Practice Sheet.

Pause for students to work.

What was the additional fee for the taxi ride? (\$5)

Write and have students write “5” in the box labeled “Initial Fee.”

Write “5” in the box labeled “Initial Fee.”

Pause for students to work. Point to each of the boxes in the Write an Equation section as you say the equation that represents the relationship in problem 2.

The equation that represents this relationship is $t = 0.50m + 5$. Write this equation under the boxes.

What is the constant term in the equation? (+5)

How do you know? *(there are no variables, the value does not change)*

Fill in and have students fill in labels for the columns in the table with the independent variable in the first column and the dependent variable in the last column in the Make a Table Section of the *Demonstration Practice Sheet*.

Now, in the Make a Table section, we need to label the columns before we can create a table of this relationship. The independent variable goes into the first column. What is the independent variable? (m)

Write and have students write the independent variable, m , in the first column of the table in the Make a Table section.

In the first column, write “ m ” for the number of miles driven.

Pause for students to work.

We will write the dependent variable in the last column. What is the dependent variable? (t)

Write and have students write the dependent variable “ t ” in the last column of the table.

In the last column, write “ t ” for the total cost of the taxi ride.

Guide students through filling out the Process column by giving values for the independent variable. Write and have students write in the Process column the mathematical operations to result in the dependent column value.

The Process column helps us determine how much the taxi will cost (output), based on how many miles are driven (input). We will use this column to keep track of our calculations.

Write “1” in the miles column and “ $0.50(1) + 5$ ” in the Process column.

Remember the relationship is \$0.50 per mile, plus an additional fee of \$5. If a taxi drives 1 mile, how would we calculate the total taxi cost? (0.50 times 1 plus 5)

Write “ $0.50(1) + 5$ ” in the Process column. What is $0.50(1) + 5$? (5.50)

Write “5.50” in the last column of the table for the dependent value.

If I ride in a taxi for 1 mile, the cost is \$5.50. Write “5.50” in the column for t .

Teacher Note

Allow students to use a calculator for computations as necessary. It is important that the students understand the concept and do not get lost in the calculations.

Continue this process by having students supply input values for the number of miles driven, calculate the total cost of the taxi ride, and record their calculations in the Process column on the *Demonstration Practice Sheet*.

3. Repeat the process outlined above for defining and describing the variables, writing the equation, and creating the table for problem 3 on the *Demonstration Practice Sheet*.

Walk students through the thought process, using specific questions such as the following to elicit verbal responses from students and check for their understanding:

- What is the first step when working with word problems?
- How should we describe the variable?
- Which variable is independent? How do you know?
- How do you know [insert variable] is dependent?
- How did you create that equation?
- How do you create the table?
- Does the process column match the equation?

Teacher Note

Extension: Having students find the common difference from the table and relating the value to the equation can generate additional discussions and connections.

4. Provide a summary of the content and process used in the lesson.

To review the key ideas of the lesson, ask students questions about the quantities that vary, the independent variable, and the dependent variable.

From the word problems that we read today, what is an example of a quantity that varies? *(answers will vary)*

Give examples of an independent variable and a dependent variable? *(answers will vary)*

How can you tell which quantity depended on which? *(answers will vary)*

Practice

Pair Practice

1. Have students work with a partner to answer the problems on the *Practice Sheet*.
2. Have student pairs define variables, fill out the table, and select the equation that best represents the situation. Tell students to be ready to justify their reasoning. Encourage students to use correct mathematical language.
3. Have student pairs present their reasoning by explaining their answers to the class. Encourage students to use correct mathematical language. Ask probing questions, such as the following, to elicit detailed responses:
 - What are the unknown quantities? How do you know?
 - Which variable is independent and which is dependent? How do you know?
 - How do you know that this equation represents this situation?
 - How do you know that this table represents this situation?

Error Correction Practice

1. Have students examine the solution strategy presented and determine why the strategy is incorrect. Have students justify why the solution strategy that they chose is incorrect.
2. Have students share the reasoning for their answers with the class.

Independent Practice

1. Have students match each contextual situation on the *Independent Practice Sheet* to the equation that best describes it.
2. Have students share their answers with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total correct at the top of the page.

Closure

Review the key ideas. Have students explain independent and dependent variables. Give students time to think of a response to the following, based on the examples from the lesson.

- Give a real-life example of 2 quantities that vary.
- Which of these is the independent variable?
- Which of these is the dependent variable?

Module 1: Variables**Lesson 12****Lesson 12: Variables as Quantities That Vary – In Context, Part III**

Lesson Objectives	<p>Students will use variables to translate verbal descriptions into equations and tables representing contextual situations.</p> <p>Students will create and use representations to organize, record, and translate contextual situations into equations.</p>	
Vocabulary	No new words are introduced.	
Reviewed Vocabulary	dependent, independent, input, output, quantities that vary, unit rate, variable	
Instructional Materials	Teacher	Student
	<ul style="list-style-type: none"> Teacher Masters (pp. 127-140) Overhead/document projector Colored pencils/markers 	<ul style="list-style-type: none"> Student Booklet (pp. 69-75) Colored pencils/markers

Cumulative Review

Have students answer the review problems independently on the *Cumulative Review Practice Sheet*. Discuss students' responses from the *Cumulative Review Practice Sheet* as part of Engage Prior/Informal Knowledge.

Engage Prior/Informal Knowledge

To open the lesson, lead a discussion to activate students' background knowledge and prerequisite skills about translating word problems into equations and tables.

Discuss problem 1 from the *Cumulative Review Practice Sheet*.

Problem 1 asks you to select the equation that represents the relationship in the word problem. On which equation did you model the relationship? ($c = 15h$)

How did you know? (\$15 per hour tells you to multiply 15 by h , the number of hours, to get the total cost, c)

Discuss problem 2 from the *Cumulative Review Practice Sheet*.

For problem 2, you were to select the equation that represents the word problem.

What equation did you select? ($c = 6p + 65$)

Why did you choose the equation $c = 6p + 65$? (\$6 per person tells you to multiply 6, the unit rate, by the number of people, " p ," then add a rental fee of \$65 to get the total)

Discuss problem 3 from the *Cumulative Review Practice Sheet*.

For problem 3, you were to select the table that represents the word problem.

Which table did you select? (A)

How did you know that table "A" was the correct representation? (answers may vary; I used the equation from

problem 2 to generate the table, if one person goes to the party then it is $6 + 65 = 71$)

Preview

This lesson will build on students' knowledge of variables in contextual situations. Students will define and use variables as quantities that vary, write equations, and create tables to represent contextual situations.

Today, we will continue working with contextual situations, or word problems. We will define variables and what they represent in situations. Then, we will create equations and tables to represent situations.

Demonstrate

1. Guide students through defining the variables, generating the table, and writing the equation for the contextual situation in problem 1 on the *Demonstration Practice Sheet*.

Write and have students write on the *Demonstration Practice Sheet* throughout the lesson to model the problem-solving process. Display the *Demonstration Practice Sheet* using an overhead or document projector for students to see.

Look at the Word Problem section in problem 1 on the Demonstration Practice Sheet. What is the first step when working with a word problem? *(read the problem and underline the critical/important information)*

Teacher Note

It may be helpful to use different colored markers or pens when identifying the variables and critical information in the contextual situation.

Read problem 1 with students and have students identify the critical information.

Follow along as I read the word problem for problem 1.
“Margret is selling cups of lemonade to earn money. She already has \$5 and is selling the lemonade for \$1.50 per cup. Find the total amount of money Margret has.” What should we underline as critical information? (*already has \$5, \$1.50 per cup*)

Underline and have students underline “already has \$5” and “\$1.50 per cup” on the *Demonstration Practice Sheet*.

Underline “already has \$5” and “\$1.50 per cup” in the word problem for problem 1.

Pause for students to work.

The words “already has” imply an initial amount. How do we translate “already has \$5” in mathematics? (+ 5)

Write and instruct students to write “+5” above the words “already has \$5.”

Write “+ 5” above the words “already has \$5”.

What does “\$1.50 per cup” represent in this problem? (*unit rate, relationship between the quantities that vary*)

How do you know? (*“per” indicates a unit rate, ratio, relationship between 2 quantities*)

Identify the quantities that vary in the word problem for problem 1. Determine the variables and describe what they represent. Write and have students write each variable and what they represent in the Define Variable(s) section on the *Demonstration Practice Sheet*.

Now, look at the Define Variable(s) section for problem 1. We need to define the 2 quantities that vary and use variables to represent them. What are the 2 quantities that vary? (*total amount of money and number of cups sold*)

Remember, when we choose letters to represent values for a situation, it is helpful to use letters that might remind us of what they represent.

Watch For



Students may assume that some variables are letters that are used as abbreviations. For example, they may choose the variable “ c ” to represent Charlie instead of the value of Charlie’s age in years. When defining variables in contextual situations, emphasize the value or quantity of the variable.

What variable should we use to represent the number of cups sold? *(answers will vary, but this script uses c for the number of cups sold)*

Write and have students write “ c ” in the first blank in the Define Variable(s) section for problem 1.

We will use the variable c to represent the number of cups sold. Write “ c ” and “the number of cups sold” in the space provided in the Define Variable(s) section.

Pause for students to work.

What variable should we use to represent Margret’s total amount of money? *(answers will vary, but this script uses t for Margret’s total amount of money)*

Write and have students write “ t ” in the second blank in the Define Variable(s) section for problem 1.

We will use the variable t to represent the total amount of money Margret has. Write, “ t ” and “total amount of money” in the space provided in the Define Variable(s) section.

Pause for students to work.

Determine the independent and dependent variable for the 2 quantities that vary in problem 1.

Recall that the independent variable determines the value of the dependent variable.

Margret’s total amount of money changes, depending on the number of cups of lemonade sold. So, t , Margret’s total amount of money, depends on c , the number of cups sold. The variable t is the dependent variable because it depends on c . So, the variable c is the independent variable.

Label and have students label each variable with “I” or “D” to indicate independent and dependent variable.

Next to the variable c write “I” to indicate independent variable. Write “D” next to the variable t to indicate the dependent variable.

Guide students through generating the equation that best represents the situation in the Write an Equation section.

In the Write an Equation section, we will write an equation to determine the total amount of money Margret makes for any number of cups sold.

Use the translations from the word problem section for problem 1 to write an equation in the Write an Equation section on the *Demonstration Practice Sheet*.

We will use the variables, c and t , to write the equation.

Look at the translations we wrote in the Word Problem section. We must multiply 1.50 by the number of cups sold. What variable did we use to represent the number of cups sold? (c)

How do we write \$1.50 per cup, mathematically? ($1.50c$)

Write and have students write “ $1.50c$ ” in the space provided in the Write an Equation section.

Write “ $1.50c$ ” in the space provided in the Write an Equation section.

Pause for students to write.

In the problem, she already has \$5. How do we represent this in our equation? (+5)

So, putting everything together, Margret earns \$1.50 per cup plus the 5 dollars she already has. This will give us the total amount of money that Margret has saved. What variable did we use to represent the total amount of money? (t)

Using “1.50 c ,” “ t ,” and “+ 5,” how do we write the equation for this word problem? ($t = 1.50c + 5$)

The equation that represents this relationship is $1.50c + 5 = t$. Write this equation in the space provided in the Write an Equation section.

Fill in and have students fill in the labels for the columns in the table with the independent variable in the first column and the dependent variable in the last column.

In the Make a Table section, we will generate a list of input and output values that represent the relationship between the number of cups sold (input) and the amount of money (output). First, we need to label the columns before we can create a table of this relationship. What variable should be in the first column? (*independent variable, c*)

What is the independent variable? (c)

How do you know? (*answers will vary; the amount of money depends on the number of cups sold*)

Write and have students write the independent variable, c , in the first column of the table in the Make a Table section.

In the first column, write “ c ” for the number of cups sold.

We will write the dependent variable in the last column. What is the dependent variable? (t)

Write and have students write the dependent variable, t , in the last column.

In the last column, write “ t ” for Margret’s total amount of money.

Guide students through filling out the Process column by giving values for the independent variable. Write and have students write in the Process column the mathematical operations to result in the dependent column values. Connections to the equation and what is written in the Process column should be made explicit to students.

The Process column helps us determine how much money Margret has earned, based on the number of cups sold. We will use this column to keep track of our calculations.

Write “2” in the column for c and “ $1.50(2) + 5$ ” in the Process column. Have students do the same.

Remember that Margret starts off with \$5 and then earns \$1.50 per cup. Write “2” in the column for c . If she were to sell 2 cups of lemonade, her total amount of money would be \$1.50 times 2, plus the \$5 she already had.

How can we write “\$1.50 times 2, plus \$5,” mathematically?
($1.50(2) + 5$)

Write “ $1.50(2) + 5$ ” in the Process column. What is the result of $1.50(2) + 5$? *(8.00)*

Write and have students write “8.00” in the column for t .

If Margret sells 2 cups of lemonade, she will have a total of \$8.00. Write “8.00” in the column for t .

Does the Process column match the equation we wrote in the Write an Equation section? *(yes)*

How do you know? *(answers will vary; both multiply the number of cups by 1.50 and add 5)*

Teacher Note

Allow students to use a calculator for computations as necessary. It is important that the students understand the concept and do not get lost in the calculations.

Continue this process by having students supply values for the number of months, calculate the total amount of money saved, and record their calculations in the Process column.

2. Guide students through defining variables, writing the equation, and generating the table for the contextual situation in problem 2 on the *Demonstration Practice Sheet*.

Look at the Word Problem section in problem 2 on the Demonstration Practice Sheet. What is the first step when working with a word problem? (*read the problem and underline the critical/important information*)

Read problem 2 to students and have students underline the critical information.

Follow along as I read the word problem for problem 2. “Lilia wants to rent a motor scooter. To rent a scooter it costs \$0.30 per mile plus an initial fee of \$45. Find the total cost to rent the scooter.” What is the critical information that we underline in this problem? (*plus initial fee of \$45, \$0.30 per mile*)

Underline and have students underline the words, “plus initial fee of \$45” and “\$0.30 per mile” on the *Demonstration Practice Sheet*.

Underline the words, “plus an initial fee of \$45” and “\$0.30 per mile” in the word problem for problem 2.

Pause for students to work.

What does “\$0.30 per mile” represent in this problem? (*unit rate, relationship between the quantities that vary*)

How do we use the unit rate “0.30 per mile” in this problem?
(multiply the number of miles by 0.30)

Write and have students write “+ 45” above the words “plus an initial fee of \$45.”

How do you translate “plus an initial fee of \$45” in mathematics? (+ 45)

Write “+ 45” above the words “plus an initial fee of \$45”.

Pause for students to work. Identify the quantities that vary. Determine the variables and describe what they represent. Write and have students write each variable and what they represent in the Define Variable(s) section on the *Demonstration Practice Sheet*.

Look at the Define Variable(s) section for problem 2. We need to define the 2 quantities that vary and represent them with variables. What are the two quantities that vary? *(the number of miles driven and the total rental cost)*

What variable should we use to represent the total cost of the rental? *(answers will vary, but this script uses t for the total cost of the taxi ride)*

Write and have students write “ t ” in the first blank in the Define Variable(s) section for problem 2.

We will use the variable t . Write “ t ” and “the total cost of the rental” in the space provided.

Pause for students to work.

What variable should we use to represent the number of miles driven? *(answers will vary, but this script uses m for the number of miles driven)*

Write and have students write “ m ” in the second blank in the Define Variable(s) section for problem 2.

We will use the variable m . Write “ m ” and “number of miles driven” in the space provided.

Have students think about which variable is independent and which is dependent for the 2 quantities that vary in problem 2. Have students Think-Pair-Share to begin the discussion.

Think about which variable is the independent variable and which is the dependent variable. Ask yourself if the number of miles driven depends on the total cost of the rental or if the total cost of the rental depends on the number of miles driven.

Pause for students to think. Have them share their reasoning with a neighbor.

Pair with your neighbor to share which variable is independent and which is dependent.

Pause for students to share. Have student pairs share what they discussed.

Which variable is the independent variable and which is the dependent variable? (*m is independent and t is dependent*)

How do you know? (*answers will vary; the total cost of the rental depends on the number of miles driven*)

Label and have students label each variable with “I” or “D” to indicate independent and dependent variable.

Next to the variable m write “I” to indicate the independent variable. Write “D” next to the variable t to indicate the dependent variable.

Guide students through generating the equation that best represents the situation in problem 2. Write the equation in the Write an Equation section of the *Demonstration Practice Sheet*.

In the Write an Equation section, we will write an equation to find the cost of a rental for any number of miles driven.

What variable did we use to represent the number of miles driven? (*m*)

How much do we pay per mile? (*\$0.30 per mile*)

What should we write in mathematics to calculate the cost of driving m miles? ($0.30m$)

Write and have students write “ $0.30m$ ” in the space provided in the Write an Equation section.

Write “ $0.30m$ ” in the space provided in the Write an Equation section.

Pause for students to write.

What other information are we given in the word problem?
(there is an initial fee of \$45)

In our equation, how do we include the initial fee of \$45? (add 45 to the product of $0.30m$)

This means the \$45 is a constant term. How do we write the equation using the variables m and t ? ($0.30m + 45 = t$, $t = 0.30m + 45$)

The equation that represents this relationship is $t = 0.30m + 45$. Write this equation on the line provided in the Write an Equation section.

Pause for students to work. Fill in and have students fill in the labels for the columns in the table with the independent variable in the first column and the dependent variable in the last column.

In the Make a Table section, we will generate a list of input and output values that represent the relationship between the number of miles driven (input) and the cost of the rental (output). We need to label the columns before we can create a table of this relationship. Which variable do we use to label the first column? (m)

How do you know? (m is the independent variable)

Write and have students write the independent variable, m , in the first column of the table in the Make a Table section.

In the first column, write “ m ” for the number of miles driven.

Pause for students to work.

We will write the dependent variable in the last column. What is the dependent variable? (*t*)

Write the dependent variable, *t*, in the last column and instruct students to do the same.

In the last column, write “*t*” for the total cost of the rental.

Guide students through filling out the Process column by giving values for the independent variable.

The Process column helps us determine how much the rental will cost based on how many miles are driven. We will use this column to keep track of our calculations.

Write “5” in the miles column and “ $0.30(5) + 45$ ” in the Process column.

Write “5” in the column for *m*. If Lilia drives 5 miles (input), how much will it cost to rent the scooter (output)? (*46.50*)

Write “46.50” in the column for *t*. How did you calculate the total cost for the rental? (*multiply 0.30 by 5 then add 45*)

How do we write this in the Process column? ($0.30(5) + 45$)

Write “ $0.30(5) + 45$ ” in the Process column.

If Lilia drives 5 miles, it will cost her \$46.50 for the scooter rental.

Does the Process column match the equation we wrote in the Write an Equation section? (*yes*)

How do you know? (*answers will vary; both multiply the number of miles by 0.30 and add 45*)

Teacher Note

Allow students to use a calculator for computations as necessary. It is important that the students understand the concept and do not get lost in the calculations.

Continue this process by having students supply input values for the number of miles driven, calculate the total cost of the taxi ride, and record their calculations in the Process column on the *Demonstration Practice Sheet*.

3. Repeat the process outlined above for defining and describing the variables, writing the equation, and creating the table for problem 3.

Walk students through the thought process, using specific questions such as the following, to elicit verbal responses from students and check for their understanding:

- What is the first step when working with word problems?
- How should we describe the variable?
- Which variable is independent and which is dependent?
- How do you know?
- How did you create that equation?
- How do you create the table? How do you know?
- Does the Process column match the equation?

Teacher Note

Extension: Having students find the common difference from the table and relating the value to the equation can generate additional discussions and connections.

4. Provide a summary of the content and process used in the lesson.

To review the key ideas of the lesson, ask students questions about the quantities that vary, the independent variable, and the dependent variable.

From the word problems that we read today, what is an example of a quantity that varies? (*answers will vary*)

Give examples of an independent variable and a dependent variable? (*answers will vary*)

How can you tell which quantity depended on which? (*answers will vary*)

Practice

Pair Practice

1. Have students work with a partner to answer the problems on the *Practice Sheet*.
2. Have student pairs define variables, fill out the table, and select the equation that best represents the situation. Tell students to be ready to justify their reasoning. Encourage students to use correct mathematical language.
3. Have student pairs present their reasoning by explaining their answers to the class. Encourage students to use correct mathematical language. Ask probing questions, such as the following, to elicit detailed responses:
 - What are the unknown quantities? How do you know?
 - Which variable is independent and which is dependent? How do you know?
 - How do you know that this equation represents this situation?
 - How did you know that this table represents this situation?

Error Correction Practice

1. Have students examine the 2 solution strategies presented and determine which strategy is incorrect. Have students justify why the solution strategy that they chose is incorrect.
2. Have students share the reasoning for their answers with the class.

Independent Practice

1. Have students match each contextual situation on the *Independent Practice Sheet* to the equation that best describes it.
2. Have students share their answers with the group. Provide corrective feedback, using mathematical language from the lesson. Have students mark the total correct at the top of the page.

Closure

Review the key ideas. Have students explain independent and dependent variables. Give students time to think of a response to the following, based on the examples from the lesson.

- Give a real-life example of 2 quantities that vary.
- Which of these is the independent variable?
- Which of these is the dependent variable?